PHYSICS

CODE - 312
Self Learning Material

Author

GAURAV AGARWAL

(B.Tech., M.Sc., P.hd.)

NIOS
VISION POINT

(An ISO 9001 : 2008 Certified Institute)

Address : 287/C, Lukerganj, Allahabad-211001

Phone: 0532-3551278, Mobile: 9598307923, 7380515253

Website: www.visionpointnios.co.in, E-mail:visionpoint287@gmail.com

YouTube **Channel : VISION POINT** :NIOS Allahabad

DISCLAIMER

Published by : VISION POINT

Telephone.: 7380515253

Typeset by : **NIOS VISION POINT**

Printed at : **Guru Kripa Images Pvt. Ltd.**

Message from Co-ordinator ....

Dear Students,

We welcome you for joining & Certifying your education act from NIOS. We are not only hopeful but also sure that you all have started attaining heights of success. You will be glad to know that NIOS is the biggest open school education system in the world. It has around 5813 study centers and around 50,000 students have got admission and passed with good marks.

NIOS is one part of three (CBSE, ICSE, NIOS) National Boards in India. Indian government has permitted us to take exams into Senior & Sr. Secondary level and gives Certificates. The syllabus of NIOS is equivalent to any other National and State level Educational Board. A student passing through the NIOS exam can take admission in any School / Colleges / Universities all over India.

Successful student of NIOS are not only studying in top most Institutions of India like IIT, Delhi University, Jawaharlal Nehru University, Jamia Hamdard University, AMU, AU, BHU but in colleges of foreign countries also. They can do Engineering, Medical or chose any other streams according to their wish.

We pray for your bright future and hope that studying in NIOS will make your life successful and bright.

With best wishes from :

Co-ordinator
(National Institute of Open School)

Why we join NIOS VISION POINT?

It will not be exaggeration that in this competitive world NIOS is a boon for the students.

In IIT-JEE, Medical and other competitive examination, now-a-days, students are crushed in both schooling system and competitive examinations. School demands 75% attendance compulsory and with other hard restrictions of schools so students do not get sufficient time for their competitive examination. These students can not get admission in any standard institution at KOTA, KANPUR or big cities but tthey are forced at study in villages and small cities. As a result, they are not able to get proper guidance and admission in I.I.T., Medical or any other advanced institution of Indian although they are talented and genius live other selected students.

NIOS is an easy solution of this problem of students, Motto of NIOS 'for all and for everyone' and NIOS follow 'Principal of freedom of learning' NIOS provides freedom from class. It is class free educational system so students get not only sufficient time but freedom to appear class anywhere in India.

NIOS is a ray of home for us parents they are overburdened wih wanted end unnecessary high fee and extra-costly books of Public School. In NIOS, there is nominal registration fee and NIOS Board provides free study materials / vedio lectures and live classes to all the students. NIOS also provides free classes from time to time as well as remedical classes for all the students.

NIOS is the only board which conducts examination two times in an academic calendar April-May and Oct-Nov. In this board these is system 'Transfer of credit marks.

NIOS provides 'On demand examination' every month through out year.

Now-a-days, NIOS Board's percentile in I.I.T., and other competitive examination is better than CBSE Board marks. So its success ratio is increased by leaps and bounds.

So, Let us join NIOS and Shape your golden future.

PREFACE

Dear Readers,

It gives me immense pleasure in presenting this 'Handbook of Physics' for class XII to everyone. In this book, it has been my sincere effort to kindle a sense of exploration with the concepts in students and nudge them towards self-study. This book has been written in accordance with the latest syllabus prescribed by the NIOS board of education.

This book has come out with, slightly a new look , more content and dimension on comparing with others ,due to the following reasons :

1. Spectrum of contents comprises well - connected topics with proper flow of precise information.
2. The subject matter has been arranged in a systematic manner strictly in accordance with the prescribed syllabus.
3. The language used in the book is simple, lucid and easily understandable.
4. Important questions have been provided in every lesson. Practicing these questions after finishing the chapter, would students would be ready for exams.
5. Every lesson contains illustrative examples, along with questions from NIOS board prescribed book.
6. At the end of each chapter, summary will be given for quick revision during exam time.

Thus it is sincerely believed that the material of the book will help students in improving their confidence and to face the examination with a lighter mind ,fetching the success.
I shall feel amply rewarded for my efforts I have put in during the preparation of this book if the students and teachers find it adequate for their requirements.

Any suggestion to enhance the quality of the book will be gratefully acknowledged.

With my Best wishes -

GAURAV AGARWAL

(B.Tech., M.Sc., P.hd.)

PHYSICS

LESSON NO.	NAME OF LESSON	MODE OF ASSESSMENT	
1	Unit, Dimensions & Vectors	TMA	
2	Motion in Straight Line	TMA	
3	Laws of Motion		PE
4	Motion in Plane	TMA	
5	Gravitation	TMA	
6	Work, Energy & Power		PE
7	Motion of Rigid Body	TMA	
8	Elastic Properties of Solids	TMA	
9	Properties of Fluids		PE
10	Kinetic Theory of Gases	TMA	
11	Thermodynamics		PE
12	Heat Transfer	TMA	
13	Simple Harmonic Motion	TMA	
14	Wave Phenomena		PE
15	Electric Charge & Electric Field		PE
16	Electric Potential & Capacitors		PE
17	Electric Current		PE
18	Magnetism & Magnetic Effect of Electric Current		PE
19	Electromagnetic Induction & Alternating Current		PE
20	Reflection & Refraction of Light	TMA	
21	Dispersion & Scattering of Light		PE
22	Wave Phenomena & Light		PE
23	Optical Instruments	TMA	
24	Structure of Atom		PE
25	Dual Nature of Radiation & Matter		PE
26	Nuclei & Radioactivity		PE
27	Nuclear Fission & Fusion		PE
28	Semiconductors & Semiconducting Devices		PE
29	Applications of Semiconducting Devices		PE
30	Communication Systems	TMA	

PE = Public Examinations
TMA = Tutor Marked Assignment

CONTENT

*This book contains chapters of PE (Public Examination)

01 LAWS OF MOTION

Inertia: It is a very common observation for all of us that any book kept on our study table will not move by itself, i.e. until and unless it is acted upon by any external force, it will not change its state of rest.

To explain the reason behind this type of questions Italian scientist Galileo has defined a new physical quantity known as *inertia*. Though it was introduced by Galileo the effective use of this term and its usage for explaining the motion of the bodies was done by another reputed physicist, Sir Isaac Newton.

"Inertia is an inherent property of a body by virtue of which it cannot change its state (i.e. rest or motion) by itself."

It says that every body in the universe does have a property which is hidden in itself and because of this property the body is unable to change its state by itself, i.e. from state of rest to state of motion or vice versa or even it's direction. This inertia is of 3 types, namely : (a) inertia of rest (b) inertia of motion and (c) inertia of direction

Inertia of rest: Inertia of rest is the inability of a body by virtue of which it can't change its state of rest to state of motion. That means any body which is at rest continues to be in the state of rest only and it can't go further into state of motion by itself.

Examples:

(i) *Passengers standing or sitting loosely in a bus experience jerk in the backward direction when the bus suddenly starts moving*. This is due to the fact that when the bus suddenly starts its motion, the lower parts of the human body shares the motion but the upper part tends to remain at rest due to inertia of rest.

(ii) *When a bullet is fired into a tightly- fitted glass pane from a reasonably close range, it makes a circular hole in the glass pane*. This is due to the fact that particles of glass around the hole tend to remain at rest due to inertia of rest. So they are unable to share the fast motion of the bullet.

Inertia of motion: Inertia of motion is the inability of a body by virtue of which it can not change its state of uniform motion along a straight line to state of rest. That means any body which is in uniform motion can't come to rest by itself until and unless some external force acts on it.

Examples:

(i) *A passenger standing in a moving bus falls forward when the bus stops suddenly*. This is due to fact that the lower part of the body comes to rest along with the bus but the upper part of the body remains in a state of motion on account of "inertia of motion".

(ii) *An athlete runs for some distance before taking a long jump.* In this way, the athlete gains momentum and this inertia of motion helps him in taking longer jump.

Inertia of direction: Inertia of direction is inability of a body by virtue of which it can't change its direction by itself. This means a body moving along a straight line can't change its direction by itself, until and unless it is acted upon by any external force.

Examples:

(i) *When a running car suddenly takes a turn, the passengers experience a jerk in the outward direction.* This is because the passengers tend to maintain their original direction of motion due to inertia of direction.

(ii) *A stone tied to one end of a string is whirled in a horizontal circle. When string breaks, the stone tends to fly off tangentially along a straight line.* This is due to inertia of direction.

Note: The mass of the body is the indirect measure of the inertia of that body.

Now let us try to understand another physical quantity "*force*", with the help of which only the mechanical state of a body changes.

"Force is that which pushes or pulls the body or tends to change the state of rest or of uniform motion in a straight line."

(a) It produces or tries to produce motion in a body at rest.

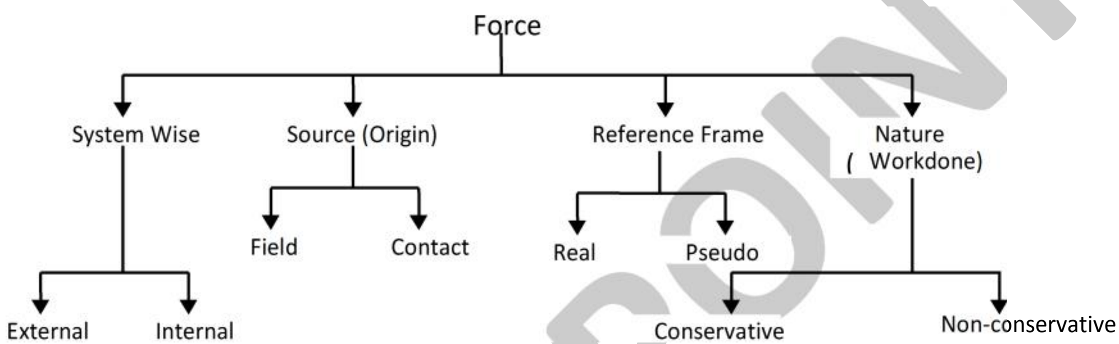
(b) It stops or tries to stop a moving body.

(c) It changes or tries to change the direction of motion of body.

(d) It produces a change in the shape of the body.

CLASSIFICATION OF FORCES

There are different types of forces in our universe. Based on the nature of the interaction between two bodies, forces may be broadly classified as under



Since we are going to encounter these forces in our analysis we will briefly discuss each force and how it acts between two bodies, its nature etc and how we are going to take it into account.

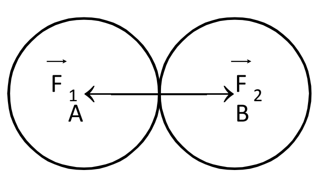
(a) Contact force: The force exerted by one surface over the surface of another body when they are physically in contact with each other is known as *contact force.*

If two surfaces that are coming into contact are perfectly smooth, then the entire contact force will act only perpendicularly (normal) to their surface of contact and it is known as "*Normal force or Normal reaction.*"

If two surfaces that are coming into contact are rough surfaces, then one component of this contact force acts perpendicular to their surface of contact and the other component of this force acts in tangential direction to their surface of contact and this component is known as "*force of friction*."

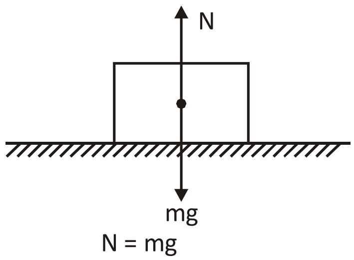
Normal Reaction, Tension, Friction, etc. are the examples of various contact forces.

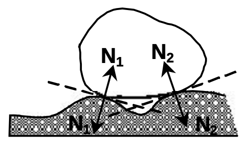
Normal reaction: The forces $\vec{F}_1$, $\vec{F}_2$ shown in the diagram acting on A and B respectively act away from the surface of contact, and prevent the two bodies from "occupying the same space".

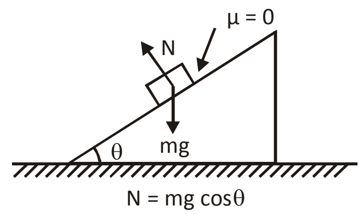


If $\vec{F}_1$ is the action, $\vec{F}_2$ is reaction: they are equal in magnitude but opposite in direction. Further, $\vec{F}_1$ and $\vec{F}_2$ are both perpendicular to the surfaces in contact and note that they act on two different bodies.

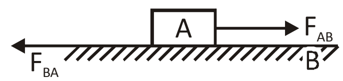
Examples:







Friction: It is a force that acts between bodies in contact with each other along the surface of contact and it opposes relative motion between the two bodies. The direction of frictional force on A is opposite to that of direction of frictional force on surface B and magnitude is same for both.

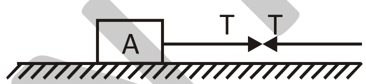


Tension (T): When a string, thread or wire is held taut, the ends of the string or thread (or wire) pull on whatever bodies are attached to them in the direction of the string. This force is known as Tension.

If the string is massless then the tension T has the same magnitude at all points throughout the string.

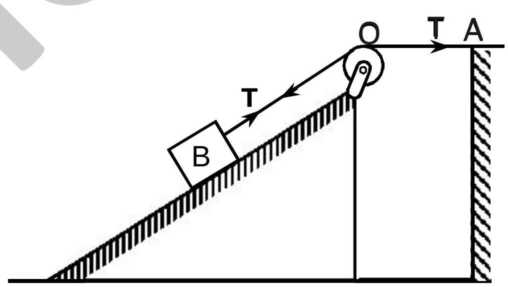
Examples:

(i) Tension in a string: For a block A pulled by a string,



(ii) The direction of tension is always away from the point of attachment to the body. In the given figure two segments of tension act at O towards A and B.

For the wedge, there are two segments of thread at the point of attachment O to the body. Hence, two tensions act on the wedge; one along OB and the other along OA.



Hence a tension acts away from the point of attachment along BO.

(b) Non-contact force: Bodies can exert forces on each other without actual physical contact. This is known as action at a distance. Such forces are known as *non-contact forces* (or) field forces, e.g. gravitational force, electrostatic forces, etc.

For the moment, we deal with actual forces. Suffice it to say that there exist pseudo-forces acting in a non-inertial frame of reference, which we will discuss later.

Forces may be conservative or non-conservative depending on whether work done against them by an external agent is recoverable or otherwise.

Newton's first law

Every body continues to be in the state of rest or of uniform motion in a straight line until and unless it is compelled to change the state of the body by an unbalanced force.

For better understanding we can divide this statement into two parts.

(i) "Every body continues to be in its state of rest until and unless some external force compels it to change the state of rest."

This part of the law is self explanatory and self evident as we come across several examples in our daily life like all inanimate objects will continue to be in the same place where they are put until they are disturbed by some external agents.

(ii) "Every body continues in its state of uniform motion in a straight line unless external force compels it to change that state."

The second part of the statement can't be readily understood as on the surface of the earth because of various types of frictional or resisting forces. For example when a ball is rolled on a horizontal surface the ball will come to halt after some time however smooth the surface may be, as we can't eliminate force of friction completely.

Momentum (Linear):

Till now we studied about inertia (translational) which is the inability of a body. Now we will study about another physical quantity called 'momentum' which is the ability of body.

Momentum is defined as the ability of a body by virtue of which it imparts or tends to impart its motion along a straight line.

Mathematically, momentum (p) is measured as the product of mass (m) and velocity (v) of the body. As velocity is a vector quantity, momentum is also a vector quantity.

$\vec{p} = m\vec{v}$ or p = mv

Unit : Its unit is kg-m/s in SI system and gm-cm/s in CGS system.

Dimensions : Its dimensions are MLT^{-1}

Illustration 1. *A block of mass 2kg is moving with a velocity of $2\hat{i} - \hat{j} + 3\hat{k}$ m/s. Find the magnitude and direction of momentum of the block with the x-axis.*

Solution: The magnitude of momentum is $2\sqrt{14}$ kg m/s and the direction is at an angle of $\tan^{-1}\sqrt{\frac{2}{7}}$ with x-axis.

Illustration 2. *A block rests on air inclined plane with enough friction to prevent it from sliding down. To start the block moving, is it easier to push it up the plane, down the plane or sideways? Why?*

Solution: It is easier to push it down the plane because we have to apply the force only to overcome from the rest part of friction force.

Newton's second law of motion

We have already studied the Newton's 1st law which has given us a qualitative idea about force. Now, we will study about Newton's IInd law which gives us a quantitative idea about force.

Whenever a cricketer catches a ball he allows a longer time for his hands to stop the ball. Otherwise the ball will hurt the cricketer. If you observe this incident carefully you can easily understand that cricketer is applying some force on ball in order to make the momentum of the body zero. And also we can understand that the magnitude of the retarding force that cricketer applies on the ball in order to stop depends on two factors.

(1) The momentum of the ball and

(2) Time for which he is applying the force

These type of observations lead Newton to state his second law of motion.

The rate of change of momentum of a body is directly proportional to the applied force and the change takes place in the direction of the force.

So for a body with constant mass,

$$\frac{d\vec{p}}{dt} \propto \vec{F} \qquad \text{or} \qquad \frac{d}{dt}(m\vec{v}) \propto \vec{F}$$

or,

$$m\frac{d\vec{v}}{dt} \propto \vec{F} \qquad ; \qquad \vec{F} = km\left(\frac{d\vec{v}}{dt}\right),$$

where k is a constant. With proper choice of units, k = 1. Thus,

$$\mathbf{\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}}$$

Unit of Force *:* Its unit is newton in SI system and dyne in CGS system.

Dimensions *:* $[MLT^{-2}]$

Illustration 3. *A body of mass m = 1 kg falls from a height h = 20 m from the ground level*

(a) What is the magnitude of total change in momentum of the body before it strikes the ground?

(b) What is the corresponding average force experienced by it? (g = 10m/sec²).

Solution: (a) Since the body falls from rest (u = 0) through a distance h before striking the ground, the speed v of the body is given by kinematics equation.

$v^2 = u^2 + 2as$; Putting a = g and s = h

we obtain $v = \sqrt{2gh}$

$\Rightarrow$ The magnitude of total change in momentum of the body

$= \Delta p = |mv - 0| = mv$, Where $v = \sqrt{2gh}$

$\Rightarrow \Delta p = m\sqrt{2gh} \Rightarrow \Delta p = (1)\sqrt{(2\times10\times20)}$ kg m/sec

$\Rightarrow \Delta p = 20$ kg m/sec.

(b) The average force experienced by the body $= F_{av} = \dfrac{\Delta p}{\Delta t}$

where Δt = time of motion of the body = t(say). We know Δp = 20 kg m/sec. Therefore we will have to find t using the given data. We know from kinematics that,

$$S = ut + \frac{1}{2}at^2 \Rightarrow h = \frac{1}{2}gt^2 \qquad (u = 0)$$

$$\Rightarrow t = \sqrt{\frac{2h}{g}} \qquad \therefore F_{av} = \frac{\Delta p}{\Delta t} = \frac{\Delta p}{t}$$

Putting the general values of ΔP and t we obtain

$$F_{av} = \frac{m\sqrt{2gh}}{\sqrt{2h/g}} = mg \qquad \Rightarrow \vec{F}_{av} = m\vec{g}\,.$$

Where mg is the weight (W) of the body and $\vec{g}$ is directed vertically downward. Therefore the body experiences a constant vertically downward force of magnitude mg.

Impulse: A large force acting for a short time to produce a finite change in momentum is called *impulse* and the force acted is called *impulsive force* or force of impulse.

Mathematically it is described as the product of force and time.

$\therefore$ Impulse (J) = F . t

$\therefore$ Impulse (J) = mv – mu and since force is variable, hence $J = \int_{t_1}^{t_2} F dt$

The area under F – t curve gives the magnitude of impulse.

Impulse is a vector quantity and its direction is same as the direction of $\vec{F}$.

Unit of Impulse : The unit in S.I. system is kgm/sec or newton -second.

Dimension : $M^1L^1T^{-1}$

Examples:

(i) *Automobiles are provided with spring shocker systems.* When the automobile bumps over an uneven road, it receives a jerk. The spring increases the time of the jerk, there by reducing the impulse of force. This minimises the damage to the vehicle.

(ii) *A man falling from a certain height receives more injuries when he falls on a marble floor than when he falls on a heap of sand.* This is because the marble floor does not yield under the weight of the man. The man is stopped abruptly. A large change of momentum takes place in a very short interval of time. But when he falls on a heap of sand, the sand yields under the weight of the man and this increases the time interval. So it reduces the force exerted by sand on man.

(iii) *It is difficult to catch a cricket ball as compared to a tennis ball moving with the same velocity.* This is because cricket ball will have more momentum than tennis ball due to its heavier mass. The change in momentum in case of cricket ball is more. Hence more force is required to stop cricket ball than tennis ball.

Illustration 4. *A cricket ball of mass 200 gm moving with velocity 15 m/s is brought to rest by a player in 0.05 sec. What is the impulse of the ball and average force exerted by player ?*

Solution: Impulse = change in momentum = m(v – u) = 0.2 (0 – 15) = – 3 Ns

Average force = Impulse / Time = $\frac{3}{0.05} = 60$ N

Newton's third law of motion

Now we have understood the qualitative and quantitative definitions of force from Newton's first and second laws. But how are the forces between two bodies related to each other if at all ? The answer is provided by the third law of motion.

Every action has an equal and opposite reaction, *which are equal in magnitude and opposite in direction.*

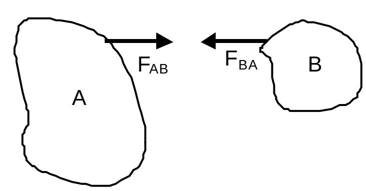
Consider two bodies A and B interacting with each other, by means of forces

$\vec{F}_{AB}$: the force exerted by body B on A

$\vec{F}_{BA}$: The force exerted by the body A on B.

According ot Newton's 3rd law : $\vec{F}_{AB} = -\vec{F}_{BA}$

(equal in magnitude & opposite in direction)

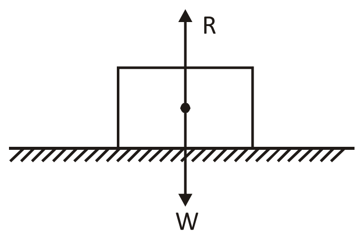


That may look fine, but it, apparently, raises a lot of questions. For example, if a horse pulls a cart and cart pulls the horse backward, how does the cart moves forward at all ?

If we observe we will find that the forces acting on the horse and the cart, though equal and opposite, they are acting not on the same body, rather, two bodies. It cannot produce equilibrium neither in horse nor in cart.

Examples:

(i) Consider a body of weight W resting on a horizontal surface. The body exerts a force (action) equal to weight W on the surface. The surface exerts a reaction R on the body in the upward direction such that W = R or in vector notation, $\vec{W} = -\vec{R}$



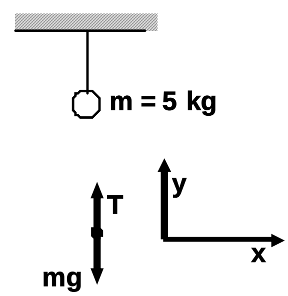
(ii) In a lawn sprinkler, when water comes out of the curved nozzles, a backward force is experienced by the sprinkler. Consequently, the sprinkler starts rotating and sprinkles water in all directions.

(iii) In order to swim, a man pushes the water backwards with his hands. As a result of the reaction offered by water to the man, the man is pushed forward.

Illustration 5. *A body of mass 5 kg is supported by a light cord. Find the tension in the cord.*

Solution: (i) The body is isolated.

(ii) The two forces acting on it are :
Tension (T) in the cord (upwards)
Weight (mg) (downwards)



(iii) Equations of equilibrium :
$\Sigma F_x = 0$
$\Sigma F_y = 0 \Rightarrow T - mg = 0$
or T = (5)(10) = 50 N

(iv) Tension in the cord equals the weight of the body.

CONSTRAINT RELATIONS

The equations showing the relation of the motions of a system of bodies, in which motion of one body is constrained by motion of other bodies, are called the constraint relations.

Applying Newton's Laws alone is not sufficient in some cases where the number of equations is less than the number of unknowns.

In the given diagram for finding the acceleration of the masses there are three unknowns, tensions T, acceleration a_1 and a_2 of masses m_1 and m_2. However we will get only two equations. Clearly Newton's laws are not sufficient to solve the problem and constraint relations provide additional equations. When the motions of bodies in a system is constrained because of pulleys, strings, wedges or other factors, we use geometry to develop additional equations.

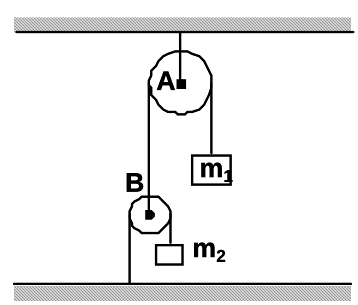
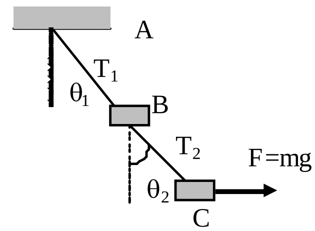


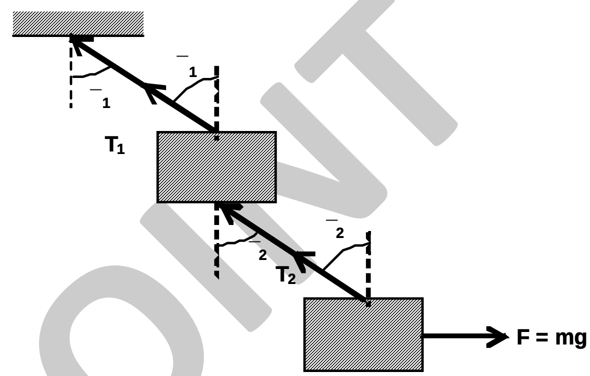
Illustration 6. *The blocks B and C in the figure have mass m each. The strings AB and BC are light, having tensions T_1 and T_2 respectively. The system is in equilibrium with a constant horizontal force mg acting on C.*



(A) $\tan\theta_1 = 1$ (B) $\tan\theta_2 = \frac{1}{2}$

(C) $T_1 = \sqrt{5}$ mg (D) $T_2 = \sqrt{5}$ mg

Solution : From F.B.D.



$T_2\cos\theta_2 = mg$

$T_2\sin\theta_2 = mg$

$\Rightarrow T_2 = \sqrt{2}\,mg$

and $\theta_2 = 45^\circ$

Also $T_1\sin\theta_1 = T_2\sin\theta_2$

and $T_2\cos\theta_2 + mg = T_1\,\mathrm{Cos}\,\theta_1$

$T_1\cos\theta_1 = mg + mg = 2\,mg$

and $T_1\sin\theta_1 = mg$

Therefore $T_1 = \sqrt{5}\,mg$

So option (C) is correct.

Application of Newton's laws of motion: techniques and approach

A separate point diagram of the body is drawn showing the different forces exerted by the bodies in the environment, this is known as free body diagram.

Application of Newton's Laws to any system (consisting of one or more objects) can be done by following a systematic method. We recommend the following steps in the order given below –

(i) Draw the complete free body diagram (FBD), showing all the forces acting on each separate body.

(ii) Select proper coordinates for analysing the motion of each body.
Include any pseudo forces within the FBD if required.

(iii) If there are any constraints, write the proper constraint equations.

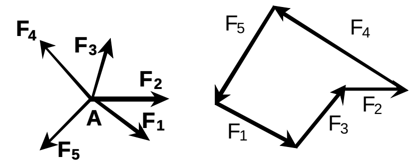
(iv) Apply Newton's 2nd law of motion : $\vec{F} = m\vec{a}$ for each body. This leads to a system of equations.

(v) Solve these equations.
(a) Identify the known and unknown quantities. Check that the number of equations equals the number of unknowns.
(b) Check the equations using dimensional analysis.
(c) After solving, check the final solution using back substitution.

(vi) If the velocity $(\vec{v})$ or position $(\vec{x})$ is required, proceed from a knowledge of acceleration ($\vec{a}$) as found from equations in step (v) and apply kinematics, e.g.

$$\frac{d\vec{v}}{dt} = \vec{a} \text{ (known) and then integrate.}$$

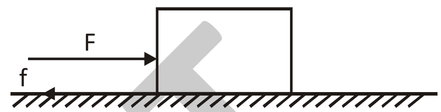
Equilibrium of concurrent forces



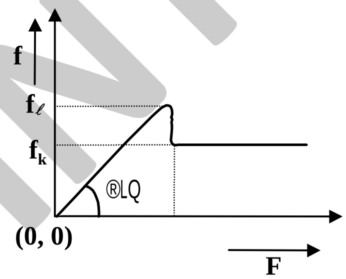
If the number of forces that are acting on a particle can be taken along the sides of any polygon both in direction as well as in magnitude, it will be in equilibrium.

Suppose that the force F_1, F_2, F_3, F_4 and F_5 are acting on the particle A and if they are in equilibrium then they will form a pentagon.

FRICTIONAL FORCE



Frictional force comes into play between two surfaces whenever there is relative motion or a tendency of relative motion between two surfaces in contact. Frictional force has the tendency to stop relative motion between the surfaces in contact.



Friction is a self-adjusting force. It changes its direction and magnitude according to the applied force or the force, which causes a tendency in the body to move. If the force increases then the opposing force also increases until the body moves beyond which it remains constant. If the applied force is plotted against the frictional force we obtain a graph as shown.

The graph shows that first frictional force increases to a certain maximum value f_ℓ with F and then suddenly decreases to a constant value f_k. For the range from 0 to f_ℓ frictional force is equal and opposite to F and hence block does not move. In this range, friction force is static.

Thus, friction can be classified as

(a) Static friction: It acts between surfaces in contact not in relative motion. It opposes the tendency of relative motion.

(b) Kinetic friction: It act acts between surfaces in contact which are in relative motion. It opposes the relative motion between the surfaces. Kinetic friction can be further classified as sliding friction and rolling friction.

Rolling friction: When a body rolls on a rough surface the frictional force developed is known as rolling friction. It is generally less than the kinetic friction or limiting friction.

Laws of static friction

Static Friction, acting between the surfaces in contact, (not in relative motion) opposes the tendency of relative motion between the surfaces.

The frictional force acts tangentially along the surfaces in contact, and the maximum value (or limiting value) of this force is proportional to the normal reaction between the two surfaces. The force of friction between two bodies is an *adjustable* force, only its maximum or limiting value is proportional to the normal reaction. Secondly, the direction of this force is determined by *all* other forces acting on the body that is by the forces that *tend* to cause relative motion. The force of static friction acts in a direction so as to *oppose* the other forces that tend to cause relative motion between the surfaces in contact.

Now, $f_{s(max)} \propto N$ where $f_l = f_{s(max)} \Rightarrow f_{s(max)} = \mu_s N$

Here μ_s = co-efficient of static friction.

N = normal reaction of the block from the surface.

$$0 \le f_s \le \mu N$$

When F exceeds f_l block starts moving and frictional force decreases to a constant value f_k. f_k is called kinetic friction and it has unique value which is given by

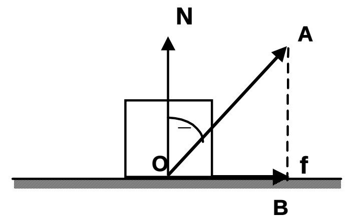
$$f_k = \mu_k N$$

Here μ_k = co-efficient of kinetic friction.

N = normal reaction.

Angle of friction: The angle made by the resultant reaction force with the vertical *(normal reaction)* is known as the angle of the friction.

Now, in the triangle OAB,



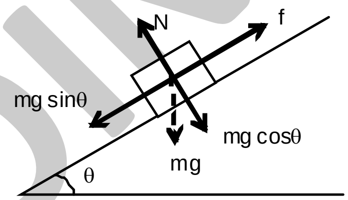
$$\frac{AB}{OB} = \cot\theta$$

$$\Rightarrow OB = AB \tan\theta$$

or, $\tan\theta = \dfrac{f}{N}$

Angle of Repose: The angle of repose is defined as the angle of the inclined plane at which a body placed on it just begins to slide. Consider an inclined plane, whose inclination with horizontal is gradually increased, till the body placed on its surface just begins to slide down, then the angle made by the plane with horizontal is called angle of repose.

From the diagram:



$f = mg \sin\theta$ . . . (i)

$N = Mg \cos\theta$

Dividing (i) by (ii)

$$\frac{f}{N} = \frac{Mg\sin\theta}{Mg\cos\theta} = \tan\theta$$

Since $\frac{f}{N} = \mu$, $\tan\theta = \mu$

Therefore, coefficient of limiting friction is equal to the tangent to the angle of repose thus angle of repose is equal to the angle of friction.

Illustration 7. *A block weighing 2 kg rests on a horizontal surface. The coefficient of static friction between the block and surface is 0.40 and kinetic friction is 0.20.*

(a) How large is the friction force acting on the block ?

(b) How large will the friction force be if a horizontal force of 5N is applied on the block?

(c) What is the minimum force that will start the block in motion?

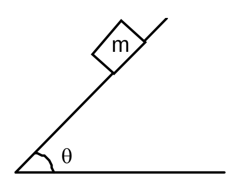
Solution: (a) As the block rests on the horizontal surface and no other force parallel to the surface is on the block, the friction force is zero.

(b) With the applied force parallel to the surfaces in contact 5 N, opposing friction becomes equal and opposite. Further the limiting friction is $\mu_s N = \mu_s Mg = 8$ N

$\therefore$ Force of friction is 5N.

(c)The minimum force that can start motion is the limiting one. $\mu_s N = \mu_s mg = 8$ N

Illustration 8. *A block of mass m is at rest on a rough inclined plane of inclination θ as shown in the figure*



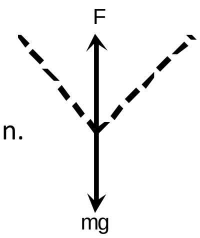
(a) Find the force exerted by the inclined plane on the block.

(b) What are the tangential and normal contact forces?

Solution: (a) The forces acting on the block are the field force $m\vec{g}$, vertically downward and total contact force $\vec{F}$ given by inclined plane on the block. As the block is at rest net force on the block is zero.

$\therefore$ $\vec{F}$ + mg = 0 [as shown in Figure]

$\therefore$ $\vec{F} = -$ mg

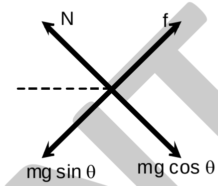


$\therefore$ The force exerted by the inclined plane on the block is $-\overrightarrow{mg}$ in vertically upward direction.

(b) The normal contact force N and tangential contact force f are shown in F.B.D. (Figure)

f = mg sin θ

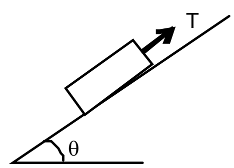
N = mg cos θ

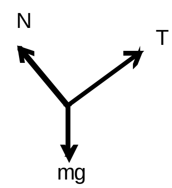


Lubrication: In some cases friction acts as a hindrance when there are moving parts in contact. A great amount of energy is lost in such type of machines like an automobile or pump or any motor. This energy converted to heat can damage the machines. So friction is reduced by suitable lubricants like oil grease, graphite etc.

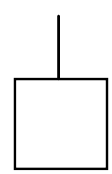
SUMMARY

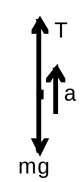
- When a body is in equilibrium in an inertial frame of reference, the vector sum of forces acting on it must be zero. Free body diagrams are essential in identifying the forces that act on the body being considered.
  Vector form
  $\Sigma\vec{F} = 0$
  Component form
  $F_x = 0,\ \Sigma F_y = 0$



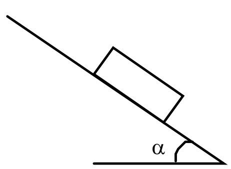


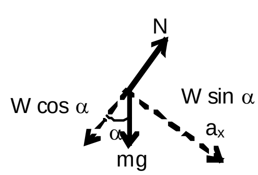
- Newton's third law is also frequently needed in equilibrium problems. The forces in an action –reaction pair never act on the same body.
- If the vector sum of forces on a body in not zero, the body accelerates. Its acceleration is given by Newton's 2nd law.
  As they are for equilibrium problems, free-body diagrams are essential for solving problems involving Newton's second law.



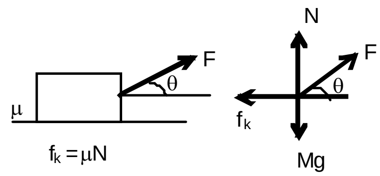


  Vector form: $\Sigma\vec{F} = m\vec{a}$
  component form $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$
- The contact force between two bodies can always be represented in terms of a normal force $\vec{n}$ perpendicular to the surface of contact and a friction force f parallel to the surface. The normal force exerted on a body by a surface is not always equal to the body's weight.

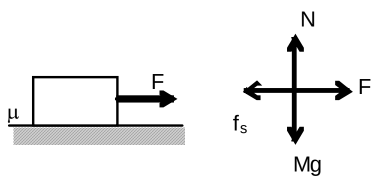




- When a body is sliding over the surface, the friction force is called kinetic friction. Its magnitude f_k is approximately equal to the normal force magnitude N multiplied by the coefficient of kinetic friction μ_k.



- When a body is not moving relative to the surface, the friction force is called static friction. The maximum possible static friction force is approximately equal to the magnitude N of the normal force multiplied by the coefficient of static friction.



- The actual static force may be anything from zero to this maximum value, depending on the situation usually μ_s is greater than μ_k for a given pair of surface in contact.

FINAL EXERCISE

1. Is it correct to state that a body always moves in the direction of the net external force acting on it?

2. What physical quantity is a measure of the inertia of a body?

3. Can a force change only the direction of velocity of an object keeping its magnitude constant?

4. State the different types of changes which a force can bring in a body when applied on it.

5. Two objects of different masses have the same momentum. Which of them is moving faster?

A boy throws up a ball with a velocity v_0. If the ball returns to the thrower with the same velocity, will there be any change in

(a) momentum of the ball?

(b) magnitude of the momentum of the ball?

7. When a ball falls from a height, its momentum increases. What causes increase in its momentum?

8. In which case will there be larger change in momentum of the object?

(a) A 150N force acts for 0.1s on a 2kg object initially at rest.

(b) A 150N force acts for 0.2s on a 2kg. object initially at rest.

9. An object is moving at a constant speed in a circular path. Does the object have constant momentum? Give reason for your answer.

10. A glass half filled with water is kept on a horizontal table in a train. Will the free surface of water remain horizontal as the train starts?

11. When a car is driven too fast around a curve it skids outwards. How would a passenger sitting inside explain the car's motion? How would an observer standing on a road explain the event?

12. What must the angular speed of the rotation of earth so that the centrifugal force makes objects fly off its surface? Take g = 10 ms^{-2}.

13. In the reference frame attached to a freely falling body of mass 2kg, what is the magnitude and direction of inertial force on the body?

02 WORK, POWER & ENERGY

Work is said to be done by a force when the point of application is displaced under the influence of the force. Work is a scalar quantity and it is measured by the product of the magnitude of force and the component of displacement along the direction of force. In fact, work is the scalar product (dot product) of the force vector and the displacement vector.

Thus, $W = \vec{F}\cdot\vec{S} = FS\cos\theta,$ where F and S are the magnitudes of force and displacement vectors and θ is the angle between them.

For $0 \le \theta \le \pi/2$, work done is positive.
For $\theta = \pi/2$, work done is zero
For $\pi/2 < \theta < 3\pi/2$, work done is negative.

For example,

(a) When a person lifts a body from the ground, the work done by the lifting force is positive but the work done by the gravitational force is negative.

(b) When a body slides on a fixed rough surface, work done by the pulling force is positive while work done by the force of friction is negative. The work done by normal reaction is zero.

Work Done By a Variable Force

The equation $W = \vec{F}\cdot\vec{S} = FS\cos\theta$ is applicable when $\vec{F}$ remains constant, but when the force is variable work is obtained by integrating $\vec{F}.\overrightarrow{dS}$

Thus, $W = \int \vec{F}\cdot\overrightarrow{dS}$

An example of a variable force is the spring force in which force depends on the extension x,

i.e. $F \propto x$

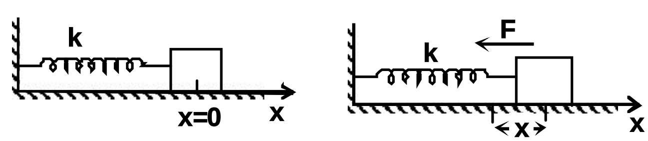
When the force is time dependent, we have,

$$W = \int \vec{F}.\overrightarrow{dx} = \int \vec{F}\cdot\left(\frac{d\vec{x}}{dt}\right)dt = \int \vec{F}.\vec{v}\ dt$$

where $\vec{F}$ and $\vec{v}$ are force and velocity vectors at an instant.

Geometrically, the work done is equal to the area between the F(x) curve and the x?axis, between the limits x_i and x_f, e.g. Consider spring force:

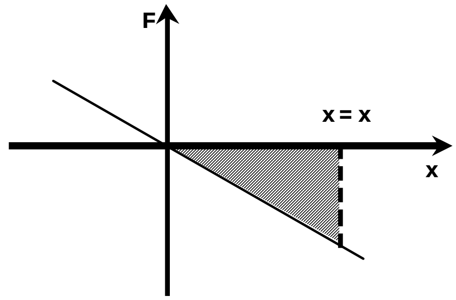
In the given figure, one end of a spring is attached to a fixed vertical support and other end to a block which can move on a horizontal frictionless table.



At x = 0, the spring is in its natural length. When the block is displaced by an amount x, a restoring force (F) due to elasticity is applied by the spring on the block.

i.e. $F = -kx$ . . . (1)

where k is the force constant of the spring which depends on the nature of the spring.



From equation (1), we can observe that

(i) F is a variable force

(ii) Work done by the force F when block is displaced from x = 0 to x is

$$W = \int_0^x F dx = \int_0^x -kx dx = -\frac{1}{2}kx^2 = \text{area under F - x graph.}$$

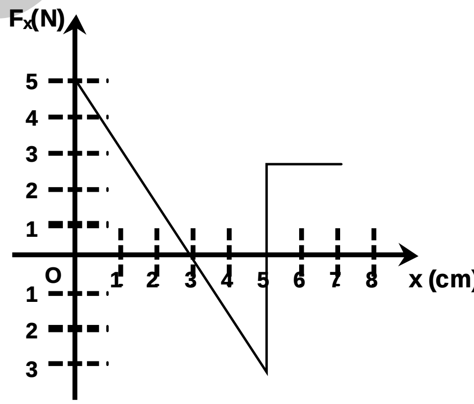
Since force (F) and displacement are opposite in direction, hence work done by restoring force is negative, but work done by the external agent is equal to the potential energy stored in the system.

Illustration 1. *The force acting along x-axis on an object as a function of x is shown in the figure. Find the work done by the force in the interval.*

(a) 0 á x-á 3 cm ,

(b) 3 á x-á 5 cm,

(c) 0 á x-á 6 cm.



Solution: We know that, work done = Area under the curve

(a) For the interval $0 \le x \le 3$ cm

$$W = \frac{1}{2} \times (0.03\,m) \times 5N = 0.075J$$

(b) For the interval $3 \le x \le 5$ cm

$$W = \frac{1}{2} \times (0.02\,m) \times 3N = -0.03J$$

(c) For the interval, $0 \le x \le 6$ cm

Work done between 5 and 6 cm = 0.01 ×3 = 0.030 J

Adding this and (a) & (b), we will get total work done in the 6 cm interval

= 0.075 – 0.030 + 0.030) = 0.075 J.

Illustration 2. *A force F = kx acting on a particle moves it from x = 0 to x = x_1. The work done in the process is*

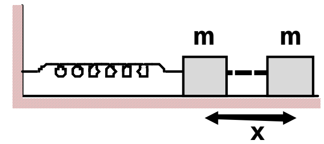
(A) kx_1^2 *(B)* $\frac{1}{2}kx_1^2$

(C) zero *(D)* kx_1^3

Solution: **(B)**

$$W = \int_0^{x_1} F.dx = \int_0^{x_1} kxdx = \frac{kx^2}{2}\Big|_0^{x_1} = \frac{kx_1^2}{2}$$

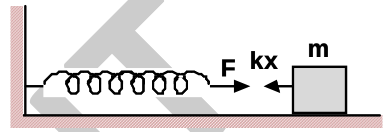
Illustration 3. *A block of mass m is attached rigidly with a light spring of force constant k. The other end of the spring is fixed to a wall. If block is displaced by a distance x, find the work done on the block by the spring for this range.*



(The spring force is given by F= –kx, where k is spring constant and x is displacement of the block from its free length.)

Solution: Since F = –kx, Therefore, force varies with displacement. This force has tendency to bring the block to its equilibrium point Hence, it is opposite to the displacement.

For infinitesimal displacement (dx) this force is supposed to be constant. Therefore, Work done by this force for the displacement dx is given by



dW = $\vec{F}.d\vec{x} = kxdx$ cos π

$\Rightarrow$ W = $\int dW = -\int_0^x kxdx \Rightarrow$ W = $-\frac{1}{2}kx^2$.

CONSERVATIVE AND NON-CONSERVATIVE FORCES

A force is said to be conservative if the work done by the force along a closed path is zero. Work done by the conservative forces depends only upon the initial and final positions and is path independent.

A force is said to be non-conservative if the work done by the force along a closed path is not zero.

Conservative forces are non-dissipative whereas non-conservative forces are dissipative.

Examples of conservative forces are gravitational force, electrostatic force, etc.

Examples of non-conservative forces are frictional force, viscous force, etc.

Mechanical Energy [Kinetic Energy + Potential Energy]

It is the capability of doing mechanical work.

Mechanical energy possessed by a body is of two types, kinetic and potential

Kinetic Energy

The capacity of a body to do work by virtue of its motion is known as kinetic energy of the body. Kinetic energy is equivalent to work done by an external force on a body of mass 'm' to bring the body from rest upto its velocity v in absence of dissipative forces.

Mathematical expression

Consider a body of mass m initially at rest. Let us consider that an external constant force F acts on the body to bring its velocity to v. If s be the displacement, then

$$v^2 = 2aS \text{ and } F = ma$$

Now, work done by the constant force, W = FS $= (ma)\left(\frac{v^2}{2a}\right) = \frac{1}{2}mv^2$

Therefore, according to the definition K.E. $= \frac{1}{2}mv^2$.

Potential Energy:

The energy possessed by a body by virtue of its position is called its potential energy.

The change in potential energy produced by a conservative force is defined as the negative of the work done by the conservative force.

$\therefore$ $U_f - U_i = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{F}.d\vec{r}$, where U_i = Potential energy at the initial reference position,

U_f = Potential energy at the final position.

Usually, the initial reference position is taken as infinity and the potential energy at infinity is assumed to be zero.

Then, we get the potential energy of a body as, $U = -\int_{\vec{r}=\infty}^{\vec{r}} \vec{F}.d\vec{r}$.

The negative derivative of the potential energy function with respect to the position gives the conservative force acting on the particle. Mathematically, $F = -\frac{dU}{dr}$.

Illustration 4. *The potential energy of a spring when stretched through a distance S is 10 J. The amount of work (in J) that must be done on this spring to stretch it through an additional distance S will be*

(A) 30 J *(B) 40 J*

(C) 10 J *(D) 20 J*

Solution: **(A).** $u_1 = \frac{1}{2}kS^2; \quad u_2 = \frac{1}{2}k(2S)^2 = 4\left\{\frac{1}{2}kS^2\right\} = 4u_1$

$$\Delta u = u_2 - u_1 = 4u_1 - u_1 = 3u_1 = 30\text{ J}$$

Illustration 5. *The momentum of a body is increased by 20%. The percentage increase in its kinetic energy is*

(A) 36% *(B) 44%*

(C) 20% *(D) 50%*

Solution: **(B)**

$$mv = mu + \frac{20}{100}mu = 1.2\,mu$$

$$\Rightarrow v = 1.2\,u$$

$$\Rightarrow v^2 = 1.44u^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}m(1.44\ u^2)$$

$$\therefore \frac{1}{2}mv^2 = 1.44(\frac{1}{2}mu^2)$$

$\therefore$ KE_{final} = 1.44 $KE_{initial}$

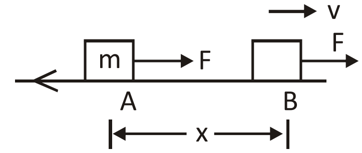
Work-Energy Theorem

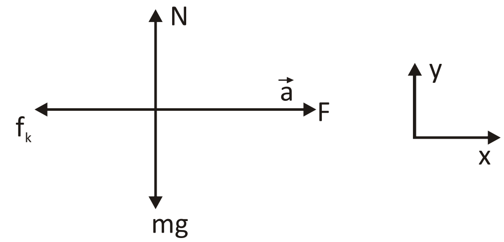
This theorem states that work done by all the forces acting on a particle or body is equal to the change in its kinetic energy.

Let us take an example shown in Figure (a), in which a block of mass m kept on a rough horizontal surface is acted upon by a constant force $\vec{F}$ parallel to the surface. The corresponding F.B.D. is shown in Fig.(b) which gives

$\vec{F} + \vec{f}_k = m\vec{a}$ ...(1)

and N = mg ...(2)





Initially, while the force $\vec{F}$ is just applied, the block is at the position A and has a velocity v_0. The force acts on it for some interval of time 't' so that the block reaches to position B at a distance x from A.

Now, the work done by the net external force is maximum along the surface and is given by

$$W = \left(\vec{F} + \vec{f}_k\right) \cdot \vec{x} \qquad \text{. . . (3)}$$

. .. . $\theta = \cos 0^0 = 1$, θ being the angle between $\vec{a}$ and $\vec{x}$.

Therefore, W = m×*a*×x . . . (4)

Again from the kinematics equation for the velocities at A and B, we have,

$$v^2 = v_0^2 + 2ax.$$

where v is the velocity of the block at position B.

Thus, $ax = \dfrac{v^2 - v_0^2}{2}$ ...(5)

Putting the value of '*a*x' from equation (4) in equation (3), we have,

$$W = m\left(\frac{v^2 - v_0^2}{2}\right)$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \qquad \text{...(6)}$$

The work done by the other two forces in F.B.D., for the displacement $\vec{x}$, are zero because $\vec{N} \cdot \vec{x} = 0$ and also $m\vec{g} \cdot \vec{x} = 0$.

Considering $\frac{1}{2}mv_0^2 = k_i$ (initial kinetic energy)

and $\frac{1}{2}mv^2 = k_f$ (final kinetic energy)

Equation (6) becomes

$W = k_f - k_i$ ...(7)

Now, equation (7) can be explained as: the net work done by all the forces on a system gives the change in kinetic energy of the system. This is known as work-energy theorem.

Thus, the change in kinetic energy of the body equals the total work done by all the forces (conservative and non conservative).

Illustration 6. *The displacement of a body in metre is a function of time according to* $x = 2t^4 + 5$. *Mass of the body is 2 kg. What is the increase in its kinetic energy one second after the start of motion?*

(A) 8 J *(B) 16 J* *(C) 32 J* *(D) 64 J*

Solution: **(D)**

$$x = 2t^4 + 5$$

$$\Rightarrow \quad v = \frac{dx}{dt} = 8t^3$$

$$a = \frac{dv}{dt} = 24t^2$$

$$\Rightarrow F = ma = 48t^2$$

$$dW = Fdx = 48t^2 \times 8t^3 dt = 48 \times 8t^5 dt$$

Increase in the kinetic energy results from the work done by the applied force

$$\Delta KE = \int_0^1 48 \times 8t^5\ dt = \frac{48 \times 8}{6} t^6 \Big|_0^1 = \frac{48 \times 8}{6} = 64\ \text{J}$$

Illustration 7. *A bullet having a speed of 153 m/s crushes through a plank of wood. After passing through the plank, its speed is 130 m/s. Another bullet, of the same mass and size but travelling at 92 m/s is fired at the plank. What will be the second bullet's speed after tunneling through? Assume that the resistance of the plank is independent of the speed of the bullet.*

Solution: Since plank does the same amount of work on the two bullets, therefore, decreases their kinetic energies equally

$$\therefore\ \frac{1}{2}m(153)^2 - \frac{1}{2}m(130)^2 = \frac{1}{2}m(92)^2 - \frac{1}{2}mv^2$$

or, $v^2 = 1955$

$\Rightarrow v = 44.2$ m/s

Conservation of Energy and Conservation of Mechanical Energy

Conservation of energy means conservation of all forms of energy together. Accounting all forms of energy within an isolated system, the total energy remains constant.

The mechanical energy accounts for only two forms of energy, namely kinetic energy, K and potential energy, U.

If only conservative forces acts on a system then total mechanical energy of the system remains constant.

i.e. K + U = const. ...(1)

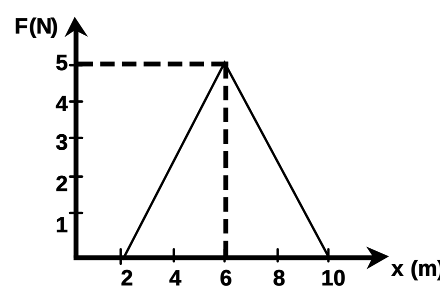
Therefore,

$\Delta K + \Delta U = 0$ ...(2)

or, $\Delta K = -\Delta U$ ...(3)

Illustration 8. *A 2 kg block is placed on a frictionless horizontal surface. A force shown in the F-x graph is applied to the block horizontally. The change in kinetic energy is :*

(A) 15 J (B) 20 J (C) 25 J (D) 30 J



Solution : **(B)** Work done = Area under F-x graph

W = 1/2 × (10 – 2) × 5 = 20 J

Work done = change in kinetic energy = 20 J

Illustration 9. *A body dropped from height h acquires momentum p just as it strikes the ground. What is the mass of the body ?*

Solution: By conservation of energy, $mgh = \frac{1}{2}mV^2$

$\Rightarrow \quad 2m^2gh = p^2$

$$\Rightarrow \quad m = \sqrt{\frac{p^2}{2gh}} = \frac{p}{\sqrt{2gh}}$$

Illustration 10. *A 40 g body starting from rest falls through a vertical distance of 25 cm before it strikes to the ground. What is the*

(a) kinetic energy of the body just before it hits the ground?

(b) velocity of the body just before it hits the ground? ($g = 9.8\ m/s^2$)

Solution: (a) As the body falls, its gravitational potential energy is converted in kinetic energy

$\therefore$ K.E. = P.E. = mgh = (0.040 kg)($9.8\ m/s^2$) $\times$ 0.25 m = 0.098 J

(b) As K.E. = $\frac{1}{2}mv^2$

$$\therefore \quad v^2 = \frac{2 \times 0.098}{0.040}$$

$\Rightarrow$ v = 2.21 m/s

$$x = \sqrt{80}\ .\ \sqrt{\frac{1}{5}} = \sqrt{16}$$

x = 4 m.

POWER

Power is defined as the rate of doing work.

Mathematically, $P = \frac{dW}{dt}$ ...(1)

As $dW = \vec{F} \cdot d\vec{x}$

Therefore, $P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{x}}{dt}$

$P = \vec{F} \cdot \vec{v}$ ...(2)

If the force is variable, we calculate the average power as

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{\int_0^t P\,dt}{\int_0^t dt} \quad ...(3)$$

Power can also be expressed as the rate of change of kinetic energy.

Let a body of mass m moves with a velocity v. Then, its kinetic energy is,

$$K = \frac{1}{2}mv^2$$

Now, $\frac{dK}{dt} = \frac{1}{2}\frac{d}{dt}\left(mv^2\right)$

$$= m\vec{v} \cdot \left(\frac{d\vec{v}}{dt}\right) = \vec{F}_{ext} \cdot \vec{v}$$

Therefore, $P = \frac{dK}{dt}$

Illustration 11. *An advertisement claims that a certain 1200 kg car can accelerate from rest to a speed of 25 m/s in a time of 8s. What average power must the motor produce to cause this acceleration ? (Ignore friction losses)*

Solution: The work done in accelerating the car is given by

$$W = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}(1200)[(25)^2 - 0] = 375 \text{ kJ}$$

$$\text{Power} = \frac{W}{t} = \frac{375}{8} = 46.9\,\text{kW}$$

Illustration 12. *A particle of mass m at rest is acted upon by a force P for a time t. Its kinetic energy after an interval t is*

(A) $\frac{P^2t^2}{m}$ *(B)* $\frac{P^2t^2}{2m}$ *(C)* $\frac{P^2t^2}{3m}$ *(D)* $\frac{Pt}{2m}$

Solution: **(B).** $S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(\frac{P}{m}\right)t^2$

$$\Delta KE = F.S = \frac{P^2t^2}{2m}$$

Equilibrium

As we have studied in the chapter of LOM, a body is said to be in translatory equilibrium if net force acting on the body is zero, i.e.

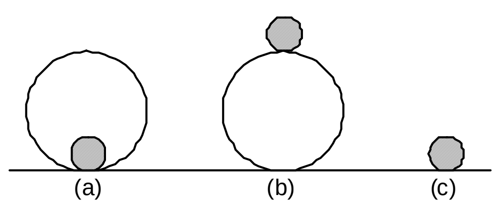
$$\vec{F}_{net} = 0$$

If the forces are conservative,

$$F = -\frac{dU}{dr}$$

and for equilibrium F = 0 $\Rightarrow \frac{dU}{dr} = 0$

i.e. at equilibrium position slope of U ? r graph is zero or the potential energy is optimum (maximum or minimum or constant). Equilibria are of three types, i.e., the situation where F = 0 and $\frac{dU}{dr} = 0$ can be obtained under three conditions. These are stable equilibrium, unstable equilibrium and neutral equilibrium. These three types of equilibrium can be better understood from the following three figures.



Three identical balls are placed in equilibrium position as shown in figures (a), (b) and (c), respectively.

In figure (a), ball is placed inside a fixed smooth spherical shell. The ball is in stable equilibrium position. In figure (b), the ball is placed over a fixed smooth sphere. This is unstable equilibrium. In figure (c), the ball is placed on a smooth horizontal ground. This ball is in neutral equilibrium position.

Table: Types of equilibrium

S. No.	Stable equilibrium	Unstable equilibrium	Neutral equilibrium
1.	$F_{net} = 0$	$F_{net} = 0$	$F_{net} = 0$
2.	$\frac{dU}{dr} = 0$ or slope of U- r graph is zero.	$\frac{dU}{dr} = 0$ or slope of U-r graph is zero.	$\frac{dU}{dr} = 0$ or slope of U-r graph is zero.
3.	When displaced from its equilibrium position, a net restoring force starts acting on the body which has a tendency to bring body back to its equilibrium position.	When displaced from its equilibrium position a net force starts acting on the body in the direction of displacement or away from the equilibrium position.	When displaced from its equilibrium position the body has neither the tendency to come back nor to move away from the original position.
4.	Potential energy in equilibrium position is minimum as compared to its neighbouring points or $\frac{d^2U}{dr^2}$ = +ve	Potential energy in equilibrium position is maximum as compared to its neighbouring points or $\frac{d^2U}{dr^2}$ = - ve	Potential energy remains constant even if the body is displaced from its. equilibrium position or $\frac{d^2U}{dr^2} = 0$
5.	When displaced from equilibrium position the centre of gravity of the body goes up.	When displaced from equilibrium position the centre of gravity of the body comes down.	When displaced from equilibrium position the centre of gravity of the body remains at the same level.

Different forms of energy

Some other forms of energies are also present in nature.

(a) Internal Energy:

Internal energy of a body is possessed because of its temperature. A body can be supposed to be made of molecules. The sum of the kinetic and potential energies of all the molecules constituting the body is called the internal energy. If the temperature of a body increases, this change cause increase in the kinetic and potential energy and Hence, in the internal energy.

(b) Heat Energy:

Due to the disordered motion of molecules of a body, it possesses heat energy.

(c) Chemical Energy:

Due to the chemical bonding of its atom, a body possesses chemical energy.

(e) Electrical Energy:

In order to move an electric charge from one point to the other in an electric field or for the transverse motion of a current carrying conductor in a magnetic field, work has to be done. This work done appears as the electrical energy of the system.

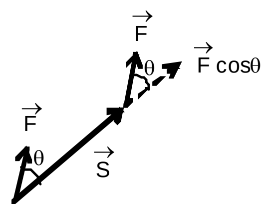
(f) Nuclear Energy

When a heavy nucleus breaks up into lighter nuclei on being bombarded by a neutron, a large amount of energy is released. This energy is called nuclear energy.

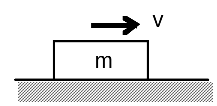
SUMMARY

- When a constant force $\vec{F}$ acts on a particle that undergoes a displacement $\vec{S}$, the work done by the force on the particle is defined as the scalar product of $\vec{F}$ and $\vec{S}$. The unit of work in SI units is Joule = Newton × meter (1J = 1 Nm). Work is a scalar quantity; It has an algebraic sign (positive or negative) but no direction in space.

  W = $\vec{F}.\vec{S}$ = F S cos θ, where θ = Angle between $\vec{F}$ and $\vec{S}$.

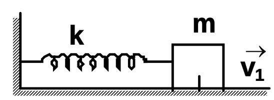


- The kinetic energy of a particle equals the amount of work required to accelerate the particle from rest to speed v. It is also equal to the amount of work a particle can do in the process of being brought to rest. Kinetic energy is a scalar quantity that has no direction in space, it is always positive or zero. Its unit is the same as the unit of work.



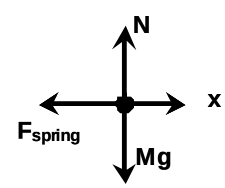
$$K = \frac{1}{2}mv^2$$

- When forces act on a particle while it undergoes a displacement, the particle's kinetic energy changes by an amount equal to the total work done on the particle by all the forces. This relation, called the work energy theorem, is valid whether the forces are constant or varying and whether the particles move along a straight line or curved path.



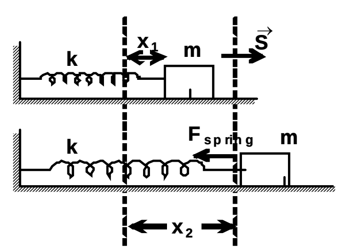
$$W_{tot} = K_2 - K_1 = \Delta K$$

- When a force varies during a straight line motion, the work done by the force is given by an integral:



$$W = \int_{x_1}^{x_2} F_x\, dx$$

- An ideal stretched or compressed spring exerts an elastic force F_x = ? kx on a particle, where x is the amount of stretch or compression. The work done by this force can be represented as a change in the elastic potential energy of the spring,



$$U = \frac{1}{2}kx^2$$

- The total potential energy is the sum of gravitational and elastic potential energy. If no forces other than the gravitational and elastic force do work on a particle, the sum of kinetic and potential energy is conserved.

$$K_1 + U_1 = K_2 + U_2$$

- When forces other than the gravitational and elastic forces do work on a particle, the work W_{other} done by these other forces equals the change in total mechanical energy, considering that the work done by gravitational and elastic forces has already been taken into account as P.E. of the system.

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

- All forces are either conservative or non-conservative. A conservative force is one for which the work-energy theorem relation is completely reversible. The work of a conservative force can always be represented by a potential energy function, but the work of a non-conservative force cannot.
- The work done by non-conservative forces manifests itself as changes in internal energy of bodies. The sum of kinetic, potential and internal energy is always conserved.

$$\Delta K + \Delta U + \Delta U_{int} = 0$$

- Power is the time rate of doing work. The average power (P_{avg}) is the amount of work ΔW done in time Δt divided by that time. The instantaneous power (P) is the limit of average power as Δt goes to zero. When a force $\vec{F}$ acts on a particle moving with velocity $\vec{v}$, the instantaneous power is the scaler product of $\vec{F}$ and $\vec{v}$. Like work and kinetic energy, power is also a scalar quantity.

$$P_{avg} = \frac{\Delta W}{\Delta t}$$

$$P = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \quad \text{and } P = \vec{F}.\vec{v}$$

FINAL EXERCISE

1. A block of mass 3 kg moving with a velocity 20m s^{-1} collides with a spring of force constant 1200 N m^{-1}. Calculate the maximum compression of the spring.

2. The power of an electric bulb is 60 W. Calculate the electrical energy consumed in 30 days if the bulb is lighted for 12 hours per day.

3. 1000 kg of water falls every second from a height of 120 m. The energy of this falling water is used to generate electricity. Calculate the power of the generator assuming no losses.

4. The speed of a 1200 kg car is 90 km h^{-1} on a highway. The driver applies brakes to stop the car. The car comes to rest in 3 seconds. Calculate the average power of the brakes.

5. A 400 g ball moving with speed 5 m s^{-1} has elastic head-on collision with another ball of mass 600 g initially at rest. Calculate the speed of the balls after collision.

6. A bullet of mass 10 g is fired with an initial velocity 500 m s^{-1}. It hits a 20 kg wooden block at rest and gets embedded into the block.

(a) Calculate the velocity of the block after the impact

(b) How much energy is lost in the collision?

7. A particle moving with a kinetic energy 3.6J collides with a spring of force constant 180 N m^{-1}. Calculate the maximum compression of the spring.

8. A car of mass 1000 kg is moving at a speed of 90 km h^{-1}. Brakes are applied and the car stops at a distance of 15 m from the braking point. What is the average force applied by brakes? If the car stops in 25 s after braking, calculate the average power of the brakes?

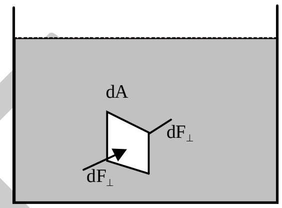
9. If an external force does 375J of work in compressing a spring, how much work is done by the spring itself?

10. Convert 10 horse power into kilowatt.

03 PROPERTIES OF FLUIDS

A fluid is a substance that can flow, so the term fluid includes both liquids and gases. Fluids differ from solids in being unable to support a shear stress.

Fluid Statics



Fluid pressure: Consider an elemental area dA inside a fluid, the fluid on one side of area presses the fluid on the other side and vice-versa. We define the pressure p at that point as the normal force per unit area.

If the pressure is same at all the points of a finite plane surface with area A, then

$p = \frac{F_{\perp}}{A}$; where $F_{\wedge}$ is the normal force on one side of the surface.

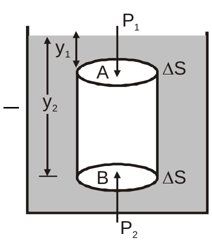
The SI unit of pressure is the 'Pascal 'where 1 Pascal = 1 Pa = 1 N/m^2

Atmospheric Pressure: It is the pressure of the earth's atmosphere. Normal atmospheric pressure at sea level (an average value) is 1 atmosphere (atm) that is equal to 1.013 ´ 10^5 Pa.

Fluid force acts perpendicular to any surface in the fluid, no matter how that surface is oriented. Hence, pressure, has no intrinsic direction of its own, it is a scalar.

The excess pressure above atmospheric pressure is called gauge pressure, and total pressure is called absolute pressure.

Variation of Pressure with Depth:



(i) Let pressure at A is P_1 and pressure at B is P_2

Then, $p_2\,\Delta S = p_1\,\Delta S + \rho g \Delta S\,(y_2 -$

or $p_2 = p_1 + \rho g(y_2 - y_1)$

Pressure increases with depth.

i.e. $\frac{dp}{dy} = \rho g$, where = density of the fluid

(ii) Pressure is same at two points in the same horizontal level.

As body is in equilibrium, $P_1\,\Delta S = P_2\,\Delta S$

or $P_1 = P_2$

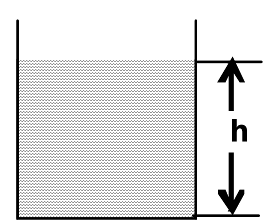
Pressure exerted by a liquid (effect of gravity)

Consider a liquid of density ρ contained in a cylinder of cross-sectional area A. Let h be the height of the liquid column. The weight of the liquid column exerts a downward thrust and hence a downward pressure.

Weight of liquid column

= Volume of liquid column × density of liquid × g

= Ah ρ g



$$\text{Pressure} = \frac{\text{weight of liquid column}}{\text{cross sectional area}} = \frac{Ah\rho g}{A} = h\rho g$$

So, the pressure exerted by a liquid column at rest is proportional to

(i) height of liquid column, and (ii) density of liquid.

Illustration 1. *Water is filled upto a height of 20 cm. The bottom of the flask is circular and has an area of 1 m^2. If the atmospheric pressure is 1.01 ò 10^5 Pa, then what force is exerted by water on the bottom? (Take g = 10 m/s^2 and density of water = 1000 kg m^{J3})*

Solution: $P = P_0 + \rho gh$

$= 1.01 \times 10^5 + 0.20 \times 1000 \times 10$

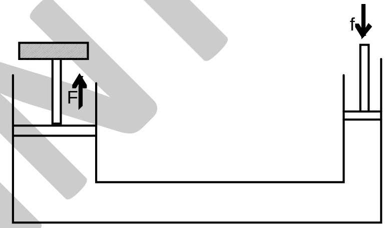
$= 1.03 \times 10^5$ Pa

$\therefore$ Force = P × A = 1.03×10^5 N

PASCAL'S LAW

A change in the pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.

HYDRAULIC LIFT



A hydraulic press is used to magnify force applied on a piston. It is based on Pascal's law. As shown in figure, a hydraulic press consists of two cylinders of different cross–sectional areas connected together.

A small force f is applied to the piston of smaller cross–sectional area 'a' and a magnified force F is produced by fluid pressure on the piston of larger cross–sectional area (A).

By Pascal's law, $\frac{f}{a} = \frac{F}{A}$

$\therefore$ $F = \frac{A}{a} f$ Hence, force F is magnified by $\frac{A}{a}$.

Illustration 2. *The diameter of the piston P_2 is 50 cm and that of the piston P_1 is 10 cm. What is the force exerted on P_2 when a force of 1 N is applied on P_1?*

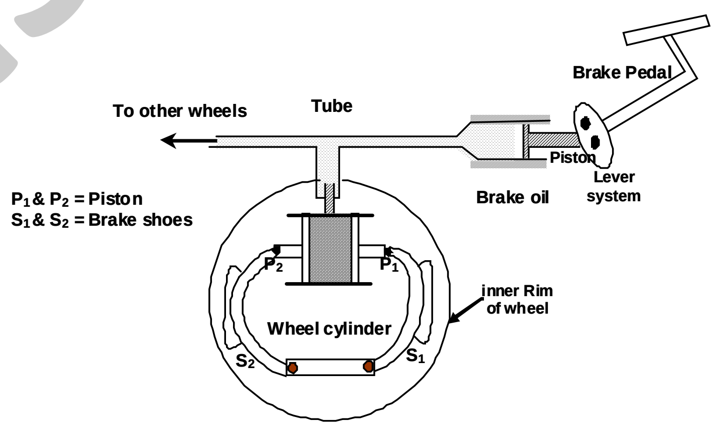
Solution: $r_2 = 25$ cm, $r_1 = 5$ cm, $F_2 = ?$

$F_1 = 1$ N

$\because \frac{F_2}{\pi r_2^2} = \frac{F_1}{\pi r_1^2}$

or $F_2 = F_1 \left(\frac{r_2}{r_1}\right)^2 = 25\text{N}$

Hydraulic Brakes: Pascal's law is used in application of hydraulic brakes also.

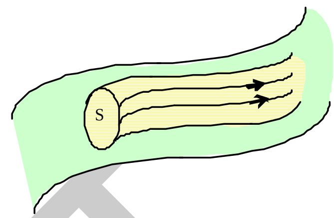


A small force is applied on the brake pedal with small area. It is magnified to a large force acting on the brakes due to larger area of the piston in brake shoes.

FLUID DYNAMICS

Steady Flow (Streamline Flow): Wherein the velocity of fluid particles reaching a particular point is the same at all times. Thus, each particle takes the same path as taken by a previous particle through that point.

Line of flow: It is the path taken by a particle in flowing liquid. In case of a steady flow, it is also called a streamline.



Tube of flow: Consider an area S in a fluid in steady flow. Draw streamlines from all the points of the periphery of S. These streamlines enclose a tube, of which S is a cross–section. No fluid enters or leaves across the surface of this tube. It is called a tube of flow.

Reynolds Number : Flow of a liquid turns from laminar to turbulent when its velocity exceeds a particular value. It is called critical velocity. Reynolds established a number which determines the nature of flow, i.e. laminar or turbulent.

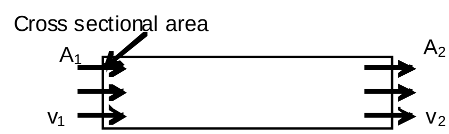
The number N = $\frac{\rho D v}{\eta}$ is called Reynolds number.

Where ρ is the density of liquid, v is its velocity, η is viscosity of the liquid and D is the diameter of the tube in which the liquid is flowing.

If N < 2000, the flow is laminar. If N is in between 2000 and 3000, the flow is unsteady and may change from laminar to turbulent and vice–versa. If N > 3000, the flow is turbulent.

Equation of continuity: *In a time* at*, the volume of liquid entering the tube of flow in a steady flow is $A_1 V_1$* at*. The same volume must flow out as the liquid is incompressible. The volume flowing out in* at *is $A_2 V_2$* at*.*

$\Rightarrow \quad A_1 V_1 = A_2 V_2$



Mass flow rate = ρAV

where ρ is the density of the liquid.

Bernoulli's Theorem

Consider a tube of flow, ABCD. In a time Δt, liquid moves and the liquid element becomes A'B'C'D'. In other words, we can also interpret that ABB'A' has gone to DCC'D'.

$$\Delta m = \rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

Work done by fluid pressure at 1 = $(P_1A_1)\, v_1 \Delta t = P_1 \Delta m/\rho$

Work done by fluid pressure at 2 = $-(P_2A_2)\, v_2 \Delta t = -P_2 \Delta m/\rho$

Work done by gravity = $-(\Delta m).g.\,(h_2 - h_1)$

Change in kinetic energy = ½ $\Delta m\, [v_2^2 - v_1^2]$

Using, work–energy theorem, (W = K),

$$P_1 \frac{\Delta m}{\rho} - P_2 \frac{\Delta m}{\rho} - \Delta m g (h_2 - h_1) = \frac{1}{2}\Delta m \left(v_2^2 - v_1^2\right)$$

$$\frac{P_1}{\rho} + g h_1 + \frac{v_1^2}{2} = \frac{P_2}{\rho} + g h_2 + \frac{v_2^2}{2}$$

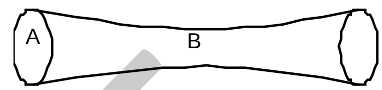
$$\Rightarrow \qquad P_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g h_2 + \frac{\rho v_2^2}{2}$$

$$P + \rho g h + \frac{\rho v^2}{2} = \text{constant}$$

In a stream-line flow of an ideal fluid, the sum of pressure energy per unit volume, potential energy per unit volume and kinetic energy per unit volume is always constant at all cross sections of the liquid.

- Bernoulli's equation is valid only for incompressible steady flow of a fluid with no viscosity.

Illustration 3. *Water flows in a horizontal tube as shown in figure. The pressure of water changes by 600 N/m² between A and B where the areas of cross*J*section are 30 cm² and 15 cm² respectively. Find the rate of flow of water through the tube.*



Solution: $\Delta P + \Delta\rho gh + \Delta\frac{1}{2}\rho v^2 = 0 \quad A_A v_A = A_B v_B$

$\Rightarrow v_B = 2v_A$

$-600 + 0 + \rho(2v_A)^2 - \rho(v_A)^2 = 0$

$\Rightarrow v_A = 0.63$ m/s

Rate of flow = (30 cm²) (0.63 m/s) = 1890 cm³ /sec.

Illustration 4. *Water enters a house through a pipe with inlet diameter of 2.0 cm at an absolute pressure of 4.0 ò 10^5 Pa (about 4 atm). A 1.0 cm diameter pipe leads to the second floor bathroom 5.0 m above. When flow speed at the inlet pipe is 1.5 m/s, find the flow speed, pressure and volume flow rate in the bathroom.*

Solution: Let points 1 and 2 be at the inlet pipe and at the bathroom. Flow speed at point 2 is obtained from continuity equation

$a_1 v_1 = a_2 v_2 \quad \Rightarrow \quad v_2 = 6.0$ m/s

Now applying Bernoulli's equation at the inlet (y=0) and at the bathroom (y_2=5.0 m)

$$P_2 = 4\times10^5 - \frac{1}{2}P\left(v_2^2 - v_1^2\right) - \rho g(y_2 - y_1)$$

Which gives $p_2 = 3.3 \times 10^5$ pa

The volume flow rate = $A_2V_2 = A_1V_1$

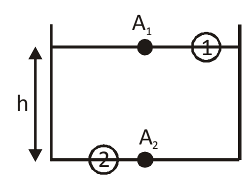
$$= \frac{\pi}{4}(0.01)^2\, 6 = 4.7\times10^{-4}\ \mathrm{m^3/s}$$

Application of Bernoulli's Equation

1. ***Speed of efflux***

Consider a tank of cross sectional area A_1 having a hole of area A_2 ($<<A_1$) at its bottom. Let us calculate the speed of water coming out from the hole.

Applying Bernoulli's equation between (1) and (2), we get



$$\frac{\rho v_1^2}{2} + \rho gh + P_0 = \frac{\rho v_2^2}{2} + 0 + P_0 \qquad ....(1)$$

By continuity equation,

$$A_1 v_1 = A_2 v_2 \qquad ....(2)$$

On solving Eqs. (1) and (2), we get

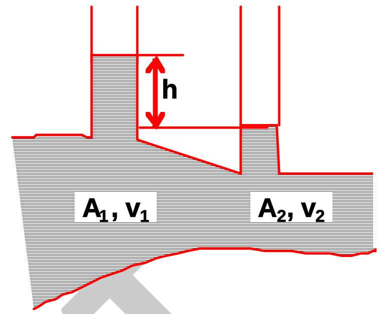
$$v_2 = \sqrt{\frac{2gh}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

2. Venturi tube

It is used to find the speed of flow of a liquid at different sections of a pipe of varying cross section.

By continuity equation, $A_1v_1 = A_2v_2$ (1)

By Bernoulli's equation,



$$P_1 + \frac{\circ\, v_1^2}{2} = P_2 + \frac{\circ\, v_2^2}{2}$$

or $$P_1 - P_2 = \frac{1}{2} \circ \left(v_1^2 - v_2^2\right) \quad (2)$$

Applying Pascal's law in the tube,

$$P_1 - P_2 = \rho gh \quad (3)$$

From Eqs. (2) and (3), we have

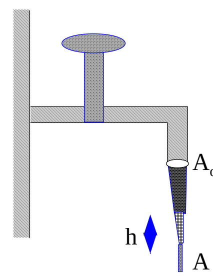
$$2gh = v_2^2 - v_1^2$$

For $A_2 >> A_1$,

$\Rightarrow$ $$v_2 = \sqrt{2gh}$$

Illustration 5. *Figure shows how the stream of water emerging from a faucet necks down as it falls. The area changes from A_o to A through a fall of h. At what rate does the water flow from the tap?*

Solution : Equation of continuity :



$A_0V_0 = AV$

Bernoulli Equation :

$$P_0 + \rho gh + \frac{1}{2}\rho v_0^2 = P_0 + \rho g(0) + \frac{1}{2}\rho v^2$$

Solving for v_0,

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$

Flow rate $R = A_0v_0 = \sqrt{\frac{2ghA^2A_0^2}{A_0^2 - A^2}} = \frac{AA_0\sqrt{2gh}}{\sqrt{A_0^2 - A^2}}$.

Note: As the jet is going into the atmosphere, the pressure at A and A_0 are equal to the atmospheric pressure.

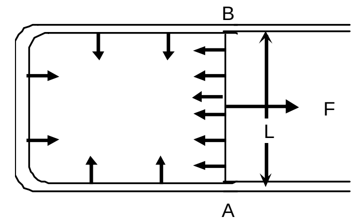
Surface Tension

The free surface of a liquid contracts so that its exposed surface area is a minimum, i.e. it behaves as if it were under tension, somewhat like a stretched elastic membrane. This property is known as surface tension. The surface tension of a liquid varies with temperature as well as dissolved impurities, etc. When is soap mixed with water, the surface tension of water decreases.

Surface tension of a liquid is measured by the normal force acting per unit length on either side of an imaginary line drawn on the free surface of the liquid. The direction of this force is perpendicular to the line and tangential to the free surface of liquid.

$$T = \frac{F}{L}$$

Consider a wire frame as shown in figure equipped with a sliding wire AB. It is dipped in soapy water. A film of liquid is formed. A force F has to be applied to hold the wire in place. Since, the soap film has two surfaces attached to the wire, the total length of the film in contact with the wire is 2L.



$$T\,(\text{Surface tension}) = \frac{F}{2L}$$

Illustration 6. *A soap film is formed on a rectangular frame of length 0.03 m dipped in a soap solution. The frame hangs from the arm of a balance. An extra mass of 2.20 ò 10^{J4} kg must be placed in the other pan to balance the pull. Calculate the surface tension of the soap solution.*

Solution : Force acting on the frame due to surface tension $F = \sigma \times \ell$

Where ℓ is the length of the frame in contact with the liquid.

Since the soap film has two surfaces

$\therefore\ \ell = 2 \times 0.03 \text{ m} = 0.06 \text{ m}$

$\therefore\ F = 0.06\ \sigma$ Newton

This must be equal to the extra weight.

$\therefore\ 0.06\ \sigma = 2.20 \times 10^{-4} \times 9.81$

or $= 0.036$ N/m

Illustration 7. *Find the work done to break a drop of water of radius 0.5 cm into identical drops of radius 1 mm. (T_{water} = 7 ò 10^{J2} N/m).*

Solution: No. of drops $= \dfrac{4/3\pi(0.5)^3}{(4/3)\pi(0.1)^3} = 125$

Surface area of big drop $= 4\pi\,(0.5)^2 \times 10^{-4} = \pi \times 10^{-4}\ \text{m}^2$

Total surface area of small drops $= 125 \times 4\pi\,(0.1)^2 \times 10^{-4} = 5\,\pi \times 10^{-4}\ \text{m}^2$

Total increase in surface area $= 4\pi \times 10^{-4}\ \text{m}^2$

$\therefore$ Work done $= T \times A = 7 \times 10^{-2}(4\pi \times 10^{-4}) = 8.8 \times 10^{-5}$ J

Properties of surface Tension

- Scalar quantity.
- Temperature sensitive.
- Impurity sensitive.
- Depends only on the nature of the liquid.
- Unit of surface tension, N/m.
- Dimension of surface tension, ML^0T^{-2}.

Surface Energy

If the area of the liquid surface has to be increased work has to be done against the force of surface tension. The work done to form a film is stored as potential energy in the surface and the amount of this energy per unit area of this surface under isothermal condition is the "intrinsic surface energy" or free surface energy density.

Work done in small displacement dx

$dW = F \times dx = 2TL\ dx$

$W = \int_0^x 2TL\ dx = 2TLx = TA$

As $A = 2Lx$ (area of both sides)

$W/A = T$ (intrinsic surface energy)

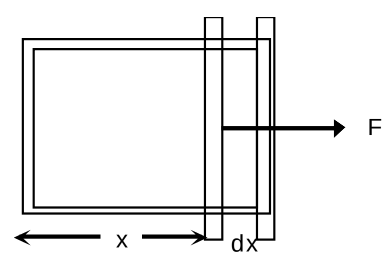


Illustration 8. *What is the surface energy of a soap bubble of radius r?*

Solution: $E = TA = T \times 4\pi r^2 \times 2$ (as it has two surfaces)

$= 8\pi r^2 T$.

Illustration 9. *What is the surface energy of an air bubble inside a soap solution?*

Solution: $E = T \times A = 4\pi r^2 T$, as it has only one surface

Illustration 10. *A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Given surface tension of liquid = ±*

Solution: If r is the radius of small drop then

$$27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left[\frac{D}{2}\right]^3 \qquad \therefore r = \frac{D}{6}$$

Initial surface area $= 4\pi\left[\frac{D}{2}\right]^2 = \pi D^2$

Final surface area = $27 \times 4\pi r^2 = 27 \times 4\pi\left[\frac{D}{6}\right]^2$

Increase in surface area = $3\pi D^2 - \pi D^2 = 3\pi D^2 - \pi D^2$

$= 2\,\pi D^2$

Increase in energy $= 2\,\pi D^2 \sigma$

Excess Pressure

The pressure inside a soap bubble and outside it, are not identical due to surface tension of the soap bubble. To calculate this pressure difference, let's first consider an air bubble inside a liquid. If the pressure difference is Dp, then the work done to increase the radius of bubble from r to (r+Dr) is given by:

$$W = F\,\Delta r = 4\,pr^2\,\Delta p\,\Delta r$$

while change in area

$$\Delta S = 4\pi\,(r + \Delta r)^2 - 4\pi r^2 = 8\,\pi r\,\Delta r$$

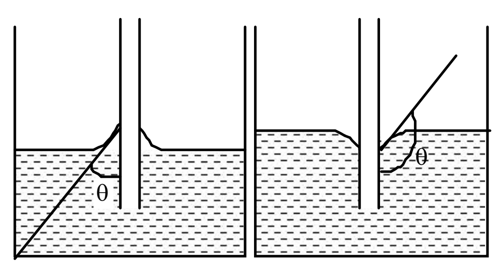
From the definition of surface tension

$$T = W/\Delta s = \frac{4® \; r^2\,\Delta p \Delta r}{8® \; r\Delta r} \qquad \Rightarrow \Delta p = \frac{2T}{r}$$

For a soap bubble in air, there are two surfaces, and so,

$$\Delta p = 2 \times 2T/\,r = \frac{4T}{r}$$

Angle of Contact



1. Angle of contact, for a solid and a liquid is defined as the angle between tangent to the liquid surface drawn at the point of contact and the solid surface inside the liquid.
2. The angle of contact of a liquid surface on a solid surface depends on the nature of the liquid and the solid.

Case I: . . . . ¯ < 90î:

The liquid surface curves up towards the solid. This happens when the force of cohesion between two liquid molecules is less than force of adhesion between the liquid and the solid.

If such a liquid is poured into a solid tube, it will have a concave meniscus. For example, a glass rod dipped in water, or water inside a glass tube.

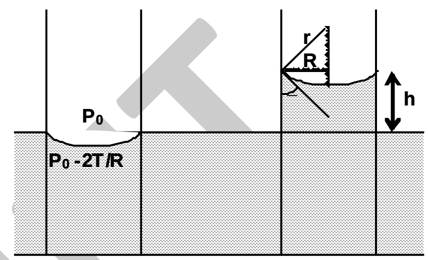
Case II : When ¯ > 90î:

The liquids surfaces get curved downward in contact with a solid. In this case the force of cohesion is greater than the force of adhesion. In such cases, solids do not get "wet". When such liquids are put into a solid tube, a convex meniscus is obtained.

For example, a glass rod dipped in mercury or mercury within a solid glass tube.

Capillarity

When a piece of chalk is dipped into water, it is observed that water rises through the pores of chalk and wets it.



Consider a glass capillary of radius R dipped in water as shown in the figure. The pressure below the meniscus will be $p_0 - \frac{2T}{r}$. To compensate for this pressure difference, water in the capillary rises so that

$$\frac{2T}{r} = \rho g h \qquad \Rightarrow h = \frac{2T}{rg\rho}$$

where r is the radius of meniscus,

and $r = \frac{R}{\cos\theta}$, where is the angle of contact and thus $h = \frac{2T\cos\theta}{R\rho g}$

If $\theta < 90°$, the meniscus will be concave, for Illustration: at a water–glass interface.

If $\theta > 90°$, the meniscus will be convex, for Illustration: at a mercury–glass interface.

Viscosity

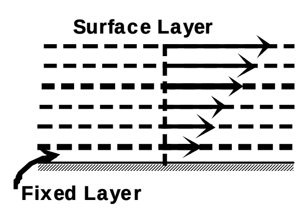
Viscosity is the property of a fluid (liquid or gas) by virtue of which it opposes the relative motion between its different layers.

Cause of viscosity:

Considering two neighbouring liquid layers A and B. Suppose A moves faster than B. B would tend to retard the motion of A. On the other hand A would try to accelerate B. Due to these two different tendencies a backward tangential force is set up. This force tends to destroy the relative motion between the two layers. This accounts for the viscous behaviour of both liquids and gases.

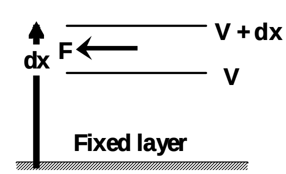
Coefficient of viscosity

Consider a liquid flowing steadily over a solid horizontal surface. The layer of the liquid in contact with the solid horizontal surface is at rest. So, this layer is a fixed layer. The velocities of other layers increase uniformly with the increase in distance from the fixed layer as shown in figure.



Consider two liquid layers P and Q at distances x and x + dx respectively from the fixed layers. Let v and v + dv be their respective velocities.

The velocity gradient $\frac{dv}{dx}$ is in a direction perpendicular to the direction of flow of the liquid.



Viscous forces F acting tangentially on a layer of the liquid is proportional to (i) the area A of the layer and (ii) the velocity gradient $\frac{dv}{dx}$

$$\therefore\ F \propto A\frac{dv}{dx}$$

or $$F = -\eta A\frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid. It depends upon the nature of the liquid. The negative sign shows that the viscous force acts in a direction opposite to the direction of the motion of the liquid

If A = 1 and $\frac{dv}{dx} = 1$,

then η = F (numerically)

The coefficient of viscosity of a liquid is the viscous force acting tangentially per unit area of a liquid layer having a per unit velocity gradient in a direction perpendicular to the direction of flow of the liquid.

The CGS unit of η is poise.

The SI unit of viscosity is equal to poise. It dimension is $ML^{-1}T^{-1}$.

Illustration 11. *A metal plate 0.04 m^2 in area is lying on a liquid layer of thickness 10^{J3}m and coefficient of viscosity 140 poise. Calculate the horizontal force needed to move the plate with a speed of 0.040 m/s.*

Solution : Area of the plate A = 0.04 m^2

Thickness $\Delta x = 10^{-3}$m

Δx is the distance of the free surface with respect to the fixed surface.

Velocity gradient, $\frac{\Delta v}{\Delta x} = \frac{0.04\ \ m/s}{10^{-3}} = 40s^{-1}$

Coefficient of viscosity, $\eta = 14kg/ms^{-1}$

Let F be the required force,

Then, $F = \eta A\frac{\Delta v}{\Delta x} = 22.4$ N

STOKE'S LAW AND TERMINAL VELOCITY

When a sphere of radius r moves with a velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is

$$F = 6\pi\eta\, rv$$

If for a sphere viscous force becomes equal to the net weight acting downward, the velocity of the body becomes constant and is known as terminal velocity.

$$6\pi\eta\, rv_T = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

where ρ and σ are densities of the sphere and the fluid, respectively.

$$\Rightarrow\quad v_T = \frac{2}{9}r^2\left\{\frac{\rho - \sigma}{\eta}\right\}g.$$

Illustration 12. *Find the viscous force on a steel ball of 2 mm radius (density 8 g/cc) that acquires a terminal velocity of 4 cm/s in falling freely in a tank of glycerine (density of glycerine 1.3 g/cc).*

Solution: $mg = 6\pi\eta rv + \frac{4}{3}\pi r^3\sigma g$

$\frac{4}{3}\pi r^3\rho_{steel}g - \frac{4}{3}\pi r^3\sigma g$ = viscous force

$= 980 \times \frac{4}{3}\times\frac{22}{7}\times\left(\frac{2}{10}\right)^3[8-1.3]$

= 220.12 dynes.

SUMMARY

- Hydrostatic pressure P at a depth h below the free surface of a liquid of density is given by P = hdg
- The upward force acting on an object submerged in a fluid is known as buoyant force.
- According to Pascal's law, when pressure is applied to any part of an enclosed liquid, it is transmitted undiminished to every point of the liquid as well as to the walls of the container.
- The liquid molecules in the liquid surface have potential energy called surface energy.
- The surface tension of a liquid may be defined as force per unit length acting on a imaginary line drawn in the surface. It is measured in Nm^{-1}.
- Surface tension of any liquid is the property by virtue of which a liquid surface acts like a stretched membrane.
- Angle of contact is defined as the angle between the tangent to the liquid surface and the wall of the container at the point of contact as measured from within the liquid.
- The liquid surface in a capillary tube is either concave or convex. This curvature is due to surface tension. The rise in capillary is given by
- Detergents are considered better cleaner of clothes because they reduce the surface tension of water-oil.
- The property of a fluid by virtue of which it opposes the relative motion between its adjacent layers is known as viscosity.
- The flow of liquid becomes turbulent when the velocity is greater than a certain value called critical velocity (vc) which depends upon the nature of the liquid and the diameter of the tube i.e. (P and d).
- Coefficient of viscosity of any liquid may be defined as the magnitude of tangential backward viscus force acting between two successive layers of unit area in contact with each other moving in a region of unit velocity gradient.

FINAL EXERCISE

1. Derive an expression for hydrostatic pressure due to a liquid column.
2. State pascal's law. Explain the working of hydraulic press.
3. Define surface tension. Find its dimensional formula.
4. Describe an experiment to show that liquid surfaces behave like a stretched membrane.
5. The hydrostatic pressure due to a liquid filled in a vessel at a depth 0.9 m is 3.0 N m^2. What will be the hydrostatic pressure at a hole in the side wall of the same vessel at a depth of 0.8 m.
6. In a hydraulic lift, how much weight is needed to lift a heavy stone of mass 1000 kg? Given the ratio of the areas of cross section of the two pistons is 5.

7. Is the work output greater than the work input? Explain.
8. What is capillary action? What are the factors on which the rise or fall of a and Fluids liquid in a capillary tube depends?
9. Why is it difficult to blow water bubbles in air while it is easier to blow soap bubble in air?
10. Why the detergents have replaced soaps to clean oily clothes.
11. Which process involves more pressure to blow a air bubble of radius 3 cm inside a soap solution or a soap bubble in air? Why?
12. Differentiate between laminar flow and turbulent flow and hence define critical velocity.
13. Define viscosity and coefficient of viscosity. Derive the units and dimensional formula of coefficient of viscosity. Which is more viscous : water or glycerine? Why?
14. What is Reynold's number? What is its significance? Define critical velocity on the basis of Reynold's number.
15. State Bernoulli's principle. Explain its application in the design of the body of an aeroplane.
16. Calculate the terminal velocity of an air bubble with 0.8 mm in diameter which rises in a liquid of viscosity of 0.15 kg m^{-1} s^{-1} and density 0.9 g m^{-3}. What will be the terminal velocity of the same bubble while rising in water? For water $\varnothing = 10^{-2}$ kg m^{-1} s^{-1}.
17. A pipe line 0.2 m in diameter, flowing full of water has a constriction of diameter 0.1 m. If the velocity in the 0.2 m pipe-line is 2 ms^{-1}. Calculate

    (i) the velocity in the constriction, and

    (ii) the discharge rate in cubic meters per second.
18. Water flows horizontally through a pipe of varying cross-section. If the pressure of water equals 5 cm of mercury at a point where the velocity of flow is 28 cm s^{-1}, then what is the pressure at another point, where the velocity of flow is 70 cm s^{-1}? [Tube density of water 1 g cm^{-3}].
19. Calculate the terminal velocity of a rain drop of radius 0.01m if the coefficient of viscosity of air is $1.8 \cdot 10^{-5}$ Ns m^{-2} and its density is 1.2 kg m^{-3}. Density of water = 1000 kg m^{-3}. Take g = 10 m s^{-2}.
20. When a liquid contained in a tumbler is stirred and placed for some time, it comes to rest, Why?
21. Calculate the radius of a capillary to have a rise of 3 cm when dipped in a vessel containing water of surface tension $7.2 \cdot 10^{-2}$ N m^{-1}. The density of water is 1000 kg m^{-3}, angle of contact is zero, and g = 10 m s^{-2}.

04 THERMODYNAMICS

System and its surroundings

SYSTEM
Any portion of universe, which is, considered as "separate", forms a "system".

There are three types of systems

OPEN
If exchange of matter and energy takes places with the surrounding.

CLOSED
If exchange of energy takes place but no exchange of matter.

ISOLATED
If no exchange of matter and energy takes place.

All those things which influence the behavior of a **'system'** are known as its **"surroundings".**

Thermodynamic parameter:

The variables which describe the state of matter e.g. pressure, volume, temperature .

Thermodynamic equilibrium:

If the variable remains unchanged with time, then the system is said to be in equilibrium.

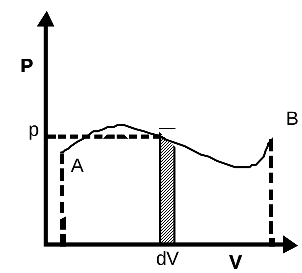
First law of thermodynamics

If ΔQ is heat given to the system and ΔW is work done by the system, then ΔU the change in internal energy can be written as

$$\Delta U = \Delta Q - \Delta W$$

This is the law of conservation of energy. It can be stated in differential form, $dQ - dW = dU$

And time rate form $\dfrac{dQ}{dt} - \dfrac{dW}{dt} = \dfrac{dU}{dt}$



Work done:

Work done by the gas (or system) over the surroundings can be calculated as

$$dW = PdV$$

$$\therefore\ W = \int_{V_i}^{V_f} PdV$$

Here dW is elemental work done by pressure P, of the system, during elemental change in volume dV.

Work done in the process AB is equal to the area under the AB curve and V-axis.

Illustration 1. *One mole of an ideal gas undergoes a cyclic change ABCD. Calculate the following from figure shown.*

(i) Work done along AB, BC, CD and DA.

(ii) Net work done in the processes.

(iii) Net change in the internal energy of the gas.

(Given 1 atm = 1.01 ò 10^5 Nm^{-2})

Solution: (i) $W_{AB} = PdV = 5 \times 1.01 \times 10^5 \times 2 \times 10^{-3}$ J = 1010 J.

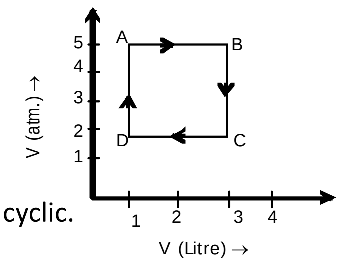
$W_{BC} = 0$

$W_{CD} = - 2 \times 1.01 \times 10^5 \times 2\ \ 10^{-3} = - 404$ J.

$W_{DA} = 0$

(ii) Net work done, W = (1010 – 404) = 606 J.

(iii) Net change in internal energy = 0. Because given process is cyclic.



Thermodynamic Processes

Quasi – Static process:

Quasi – static process is an infinitely slow process such that the system remains in equilibrium with the surroundings throughout. In this process, the pressure and temperature of the environment can differ from the system only infinitesimally.

Isobaric process

A process taking place at constant pressure is called an isobaric process. For example, the boiling of water to steam in an open pot or the freezing of water to ice taking place at a constant atmospheric pressure are isobaric processes. In this process, the work done is equal to PaV.

Isochoric process

A process taking place at constant volume is called an isochoric process, ??v = 0)

work done, $dW = p.dV = 0$

$\therefore$ Net change in internal energy, $\Delta U = Q$

Adiabatic process

When a system passes from an initial state i to a final state f through a process such that no heat flows into or out of the system, then the process is called adiabatic.

Such a process can occur when the system is thermally insulated from the surroundings, or when the process is very rapid so that there is little or no time for the heat to flow into or out of the system.

$$W = \frac{nR\left(T_i - T_f\right)}{\gamma - 1}$$

Isothermal process

When a system undergoes a process under the condition that its temperature remains constant, then the process is said to be "isothermal".

Such a process can occur when the system is contained in a perfectly conducting chamber and the process is carried out very slowly, so that there is sufficient time for heat exchange.

As the temperature of the system remains constant (T = constant), i.e. PV = constant.

$$W = nRT\ \ln\left(\frac{V_f}{V_i}\right); \quad W = nRT\ \ln\left(\frac{P_i}{P_f}\right).$$

Free expansion:

If a system (a gas), expands in such a way that no heat enters or leaves the system (adiabatic process) and also no work is done by or on the system, then the expansion is called the free expansion.

Consider an adiabatic vessel with rigid walls divided into two parts. One containing a gas and the other evacuated. When the partition is suddenly broken, the gas rushes into the vacuum and expands freely.

$\therefore$ Net change in internal energy

$U_{-f} - U_i = \Delta Q\ \ W$ as $\Delta Q = 0$ and $W = 0$

$\therefore$ $U_i = U_f$

The initial and final internal energies are equal in free expansion.

Cyclic process:

When a particular system passes through various processes such that the initial and final states are the same, then the combination of such processes is called a cyclic process.

$\Delta U = 0$ ***for a cyclic process. Since,***

$\Delta U = \Delta Q - W \quad \Rightarrow \quad 0 = \Delta Q - W$

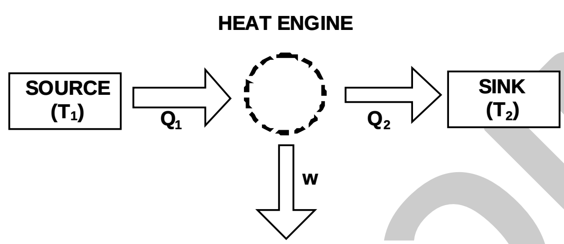
or $\Delta Q = W$, over a cyclic process.

Heat Engine

Heat engine is a device used for converting heat energy into mechanical energy. A heat engine consists essentially of the following parts:

(i) Source or Heat reservoir: It is the supplier of heat energy.

(ii) Sink or Cold reservoir: That heat which has not been converted into work is rejected to the sink



Block diagram of heat engine

(iii) Working substance: It absorbs a certain quantity of heat from the source, converts a part of it into work and rejects the remaining heat to the sink. It is taken through a cyclic operations

Efficiency: It is defined as the ratio of the net external work done by the engine during one cycle to the heat absorbed from the source during that cycle It is denoted by η.

i.e. $$\eta = \frac{W}{Q_1}$$

Where W is the net external work done by the engine during one cycle and Q_1 is the heat absorbed from the source during that cycle.

Since the working substance returns to its initial state after completing one cycle therefore there is no change in internal energy.

Therefore, by applying first law of thermodynamics, we get

$$Q_1 - Q_2 = W$$

Where Q_2 is the amount of heat rejected to the sink,

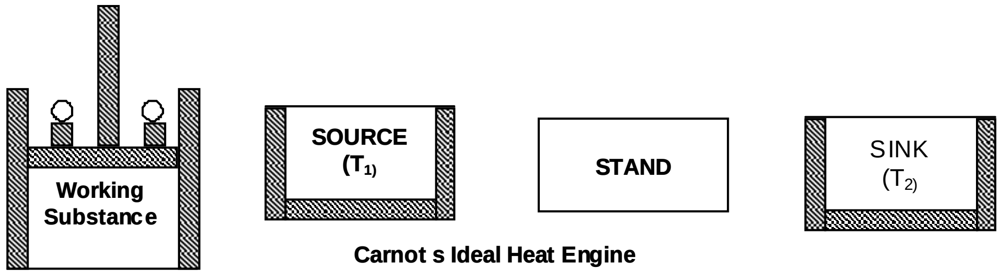
i.e. $$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

For a given value of Q_1, the smaller the value of Q_2, the higher is the efficiency of the heat engine,

Carnot's Ideal Heat Engine: It is an ideal heat engine which is free from all the imperfections of an actual engine. It consists essentially of the following parts.

(i) Source: It serves as source of heat. It is maintained at a constant high temperature T_1K It has infinite thermal capacity.



Carnot s Ideal Heat Engine

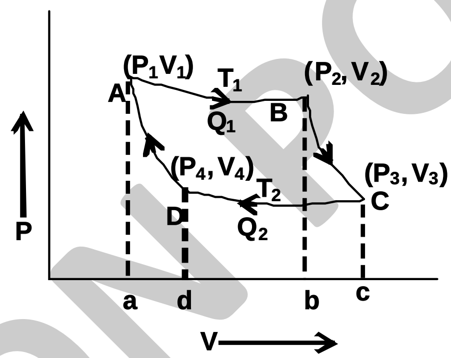
(ii) Sink: It is a cold body maintained at constant low temperature T_2 K. It also has infinite thermal capacity.

(iii) Insulating stand: It is a perfectly non – conducting pad.

(iv) Cylinder it has perfectly non-conducting wall but bottom is perfectly conducting. It is fitted with perfectly non- conducting and frictionless piston over which some weights are placed. One mole of an ideal gas is enclosed in the cylinder. This ideal gas acts as the working substance,

The working substance is subjected to the following four successive reversible operations so as to complete a reversible cycle. This cycle is called Carnot's cycle.

Let the pressure, volume and temperature be P_1, V_1, and T_1 respectively. The state of the working substance is represented by the point A in the P – V diagram.



Operation I (Isothermal Expansion):

The cylinder containing the working substance is placed in contact with the heat source. So that the gas acquires the constant temperature T_1 of the source. Now, the gas is allowed to expand slowly. Therefore, it draws heat from the source through its conducting base and the piston slowly moves out of the cylinder. Piston moves slowly to maintain the temperature at T. Thus gas expands isothermally from the initial state A (P_1, V_1) to the final state B (P_2, V_2) along the isothermal AB at temperature T_1. If Q_1 be the amount of heat absorbed by the gas from the source and W_1 be the work done by it. Then

$$Q_1 = W_1 = \int_{V_1}^{V_2} PdV = RT\ell n_e \frac{V_2}{V_1} = \qquad \text{... (i)}$$

Operation II (Adiabatic expansion):

The cylinder is now removed from the source and is placed in contact with the insulating stand. The gas is completely thermally isolated from the surroundings. The gas expands adiabatically from volume V_2 to V_3 till its temperature falls to T_2K. The pressure falls from P_2 to P_3. This expansion is represented by the curve BC in the given figure.

The work done by the gas is

$$W_2 = \int_{V_2}^{V_3} PdV = \frac{R}{\gamma - 1}(T_1 - T_2) \qquad \text{. . . (ii)}$$

Operation – III (Isothermal Compression):

Now the cylinder is placed on the sink and the gas compressed isothermally untill the pressure and volume because P_4 and V_4 respectively as shown in the PV diagram by the isothermal curve CD. The heat Q_2 developed in compression is absorbed by the sink. If W_3 be the work done on the gas then

$$Q_2 = W_3 = RT_2 \ell n_e \frac{V_3}{V_4} \qquad \text{. . . (iii)}$$

Operation IV (Adiabatic Compression):

Now, the cylinder is pleased on the insulating stand and the gas is compressed adiabatically till it attains pressure P_1, volume V_1 and temperature T_1 again. This compression is represented by the curve DA. Work done on the gas

$$W_4 = -\int_{V_2}^{V_1} PdV = \frac{R}{\gamma - 1}\left(T_1 - T_2\right) \qquad \text{. . . (iv)}$$

Net work done W by the working substance during one cycle.

W = work done by the gas – work done on the gas

$W = W_1 + W_2 - W_{3-} - W_4 = W_1 - W_3$

= area ABCDA

Efficiency of Carnot's cycle:

The efficiency of Carnot's heat engine is given by

$$\eta = 1 - \frac{Q_2}{Q_1}$$

from equation (1) & (3)

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \frac{\ell n_e\left(\frac{V_3}{V_4}\right)}{\ell n_e\left(\frac{V_2}{V_1}\right)} \qquad \text{. . . (v)}$$

The points B and C lie on the same adiabatic

$$\therefore T_1 V_1^{\gamma-1} = T_2 V_3^{\gamma-1} \qquad \text{. . . (vi)}$$

The points A and D lie on the same adiabatic

$$\therefore T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1} \qquad \text{. . . (vii)}$$

from equation (6) & (7)

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

from equation (5)
$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\therefore \eta = 1 - \frac{T_2}{T_1}$$

Therefore,

(i) Efficiency is independent of the nature of the working substance.

(ii) It depends upon the temperatures of the heat source and sink only.

Illustration 2. *During isothermal expansion at 800 K, the working substance of a Carnot's engine extracts 480 cal of heat. If the sink be at 300K, calculate*

(i) the work done by the working substance during isothermal expansion.

(ii) the work done on the substance during isothermal comparison.

(iii) the efficiency of ideal engine.

Solution: Given that $T_1 = 800$ K, $T_2 = 300$ K, $Q_1 = 480$ cal. and we know that,

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

or $$Q_2 = \frac{T_2}{T_1} \times 480\text{cal.}$$

$$Q_2 = \frac{300}{800} \times 480 \text{ cal} = 180\text{cal.}$$

(i) Work done during isothermal expansion

$$W_1 = Q_1 = 480 \text{ cal} = 480 \times 4.2 \text{ J}$$
$$= 2016 \text{ J.}$$

(ii) Work done during isothermal compression

$$Q = 180 \text{ cal} = 180 \times 4.2 \text{ J} = 756 \text{ J}$$

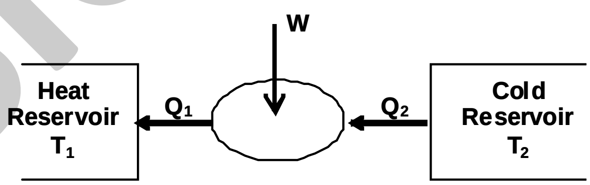
(iii) Efficiency $$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$$

$$= \left(1 - \frac{180}{480}\right) \times 100 = 62.5\ \%$$

Refrigerators or heat pumps:

A refrigerator is reverse of a heat engine. In a refrigerator working substance extracts heat Q_2 from the cold reservoir at temperature T_2. Some external work W is done on it and heat Q_1 is released to the hot reservoir at temperature T-$_{1}$-. A heat pump is the same as a refrigerator.

As heat is to be removed from the sink at lower temperature, an amount of work equal to $Q_1 - Q_2$ is performed by the compressor of the refrigerator to remove heat from sink and then to reject the total heat $Q_1 = (Q_2 + Q_1 - Q_2)$ to the source through the radiator fixed at its back.



Coefficient of performance:

It is defined as the ratio of the quantity of heat removed per cycle from the contents of the refrigerator to the work done by the external agency to remove it. It is denoted by β.

Thus $$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

$$\beta = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

For a carnat’s cycle,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \qquad \therefore \beta = \frac{1}{\frac{T_1}{T_2} - 1}$$

$$\beta = \frac{T_2}{T_1 - T_2}$$

Coefficient of performance cannot be infinite,

Reversible process:

It is that process which can be retraced in the opposite direction so that the system and the surroundings pass through exactly tHe same state at each stage as in the direct process.

e.g. If some work is done by the system in the direct process, the same amount of work is done on the system in the reverse process.

In order that a process may be reversible, it should satisfy the following conditions.

(i) The process should proceed at an extremely slow rate so that the following requirements are met.

(a) The system should remain in mechanical equilibrium,

(b) The system should remain in chemical equilibrium

(c) The system should remain in thermal equilibrium,

(ii) No dissipative forces should be present.

Irreversible process:

It is the process which is not exactly reversed, i.e. the system does not pass through the same intermediate state as in the direct process.

Every process in nature is an irreversible process. e.g.

(1) All chemical reactions are irreversible.

(2) Flow of current through a conductor is an irreversible process.

Second Law of Thermodynamics:

There are several statements of this law but the following two are the most significant.

(i) Kelvin - Plank statement:

It is impossible to construct an engine, operating in a cycle, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work.

(ii) Clausius Statement:

It is impossible to make heat flow from a body at a lower temperature to a body at a higher temperature without doing external work on the working substance.

These two statements are equivalent. The second law of thermodynamics is applicable only to a cyclic process in which the system returns to its original state after a complete cycle of changes.

The second law implies that no heat engine can have efficiency η equal to 1 or no refrigerator can have coefficient of performance α equal to infinity.

Carnot’s Theorem:

No heat engine working between given temperatures can have efficiency greater than that of a reversible engine working between the same temperatures.

SUMMARY

- Heat is a form of energy which produces in us the sensation of warmth.
- The energy which flows from a body at higher temperature to a body at lower temperature because of temperature difference is called heat energy.
- The most commonly known unit of heat energy is calorie. 1 cal = 4.18 J and
- A graph which indicates how the pressure (P) of a system varies with its volume during a thermodynamic process is known as indicator diagram.
- Work done during expansion or compression of a gas is $P\Delta V = P(V_f - V_i)$.
- Zeroth law of thermodynamics states that if two systems are separately in thermal equilibrium with a third system, then they must also be in thermal equilibrium with each other.
- The sum of kinetic energy and potential energy of the molecules of a body gives the internal energy. The relation between internal energy and work is $U_i - U_f = -W$.
- The first law of thermodynamics states that the amount of heat given to a system is equal to the sum of change in internal energy of the system and the external work done.
- First law of thermodynamics tells nothing about the direction of the process.
- The process which can be retraced in the opposite direction from its final state to initial state is called a reversible process.
- The process which can not be retraced along the same equilibrium state from final to the initial state is called an irreversible process. A process that occurs at constant temperature is an isothermal process.
- Any thermodynamic process that occurs at constant heat is an adiabatic process.
- The different states of matter are called its phase and the pressure and temperature diagram showing three phases of matter is called a phase diagram.
- Triple point is a point (on the phase diagram) at which solid, liquid and vapour states of matter can co-exist. It is characterized by a particular temperature and pressure.
- According to Kelvin-Planck's statement of second law, it is not possible to obtain a continuous supply of work from a single source of heat.
- **Zeroth Law of thermodynamics** If a body A is in thermal equilibrium with another body B and a body B is in thermal equilibrium with another body C, then A is also in thermal equilibrium with C.
- **First Law of Thermodynamics** If ΔQ is heat given to a system, and ΔW is work done by the system, then ΔU, the change in its internal energy can be written as

  $Q = \Delta U + \Delta W$, where $\Delta U = n\,C_v \Delta T$,

  $\Delta W = \int_{v}^{v_f} PdV$ = Area enclosed under P-V graph.

  $\Delta Q = nC\Delta T$, where C = specific heat capacity of a gas, for that process.
- **Important Cases of the First Law of Thermodynamics**

  (a) Isobaric process (Pressure = constant)

  $\Delta Q = \Delta U + \Delta W$

  $nC_p\Delta T = nC_v\Delta T + nR\Delta T$

  $C_p - C_v = R$ (Mayer's equation)

  (b) Isochoric process (ΔW = 0, Volume = constant)

  $\Delta Q = \Delta U = nC_v\Delta T$

(c) Isothermal process ($\Delta U = 0$; temperature = constant)

$$\Delta Q = \Delta W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \ln\left[\frac{V_2}{V_1}\right] = nRT \ln\left[\frac{P_1}{P_2}\right]$$

(d) Adiabatic process ($\Delta Q = 0$)

$TV^{\gamma-1}$ = constant

(e) Cyclic process

$\Delta U = 0$, since process returns to the same initial state

$\Delta U = \Delta Q - \Delta W = 0$

$\therefore \quad \Delta Q = \Delta W$

(f)Free expansion

In free expansion (a gas) expands in such a way that no heat enters or leaves the system (adiabatic process) and also no work is done by or on the system, then the expansion is called the free expansion

$U_f - U_i = Q - W$, Now Q = 0, W = 0

$\therefore U_f = U_i$

FINAL EXERCISE

1. (i) A Carnot engine has the same efficiency between 1000K and 500K and between TK and 1000K. Calculate T.

   (ii) A Carnot engine working between an unknown temperature T and ice point gives an efficiency of 0.68. Deduce the value of T.
2. Distinguish between the terms internal energy and heat energy.
3. What do you mean by an indicator diagram. Derive an expression for the work done during expansion of an ideal gas.
4. Define temperature using the Zeroth law of thermodynamics.
5. State the first law of thermodynamics and its limitations.
6. What is the difference between isothermal, adiabatic, isobaric and isochoric processes?
7. State the Second law of thermodynamics.
8. Discuss reversible and irreversible processes with examples.
9. Explain Carnot's cycle. Use the indicator diagram to calculate its efficiency.

05 WAVE PHENOMENA

The vehicle, which is responsible for the transmission of energy from one place to another through a medium without any bulk motion of the medium in the direction of energy flow, is called a wave.

Types of waves:

(a) Mechanical waves: The wave, produced due to the vibration of material particles of an elastic medium e.g. sound wave, vibrating string.

(b) Electro magnetic waves: Waves which are produced due to the periodic vibration of two mutually perpendicular electric and magnetic fields are Electromagnetic waves. It propagates in a direction perpendicular to both electric and magnetic field. e.g. light waves, x-ray, γ-ray etc.

Equation of a simple harmonic plane wave

In case of harmonic wave the displacement of successive particles of the medium is given by a sine function or cosine function of position.

The displacement y at t = 0 is given by

$$y = A \sin kx \qquad .....(iv)$$

Where A and k are constants.

Suppose this disturbance is propagating along positive x - direction then

$$y = A \sin k(x - vt) \qquad ......(v)$$

Since the waveform represented by equation (iv) is based on sine function, it would repeat itself at regular distances. The first repetition would take place when

$$kx = 2\pi \quad \text{or} \quad x = 2\pi/k$$

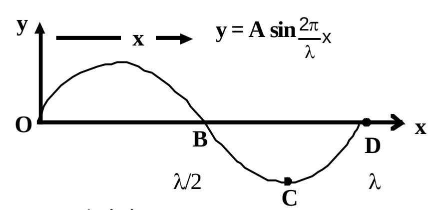
This distance after which the repetition takes place is called the wavelength and denoted by λ. Hence

$$\lambda = 2\pi/k \quad \text{or} \quad k = 2\pi/\lambda$$

This constant k is called propagation constant or wave number. Now equation (v) turns into

$$y = A \sin (x - vt) \qquad .....(vi)$$

At $\quad t = 0 \quad y = A \sin \frac{2\pi}{\lambda} x \quad .....(vii)$



Relation between wavelength and velocity of propagation

Time taken for one complete cycle of wave to pass any point is the time period (T).

This is also the time taken by the disturbance in propagating a distance λ.

$v = \ = f\lambda \qquad$ where f = frequency (Hz)

$$\omega = \frac{2\pi}{T} = 2\pi f = \text{circular frequency (rad/s)}$$

Different forms of simple harmonic wave equation:

The wave equation of a wave traveling in x-direction

$$y = A \sin \omega(t - x/v)$$

$$\Rightarrow \quad y = A \sin (\omega t - kx) = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) = A \sin k (vt - x)$$

Also, it is to be noted that we have made our particular choice of t = 0 in writing equation y = A sin ωt. The origin of time is chosen at an instant when the left end x = 0 is crossing its mean position y = 0 and is going up. For a general choice of the origin of time, we have to add a phase constant so that equation will be

$$y = A \sin [\omega (t - x / v) + \phi]$$

The constant ϕ will be $\pi/2$ if we choose t = 0 at an instant when the left end reaches its extreme position y = A. The equation will then be

$$y = A \cos \omega (t - x / v)$$

If t = 0 is taken at the instant when the left end is crossing the mean position from upward to downward direction, ϕ will be π and the equation will be

$$y = A \sin \omega (x / v - t)$$

$$y = A \sin (kx - \omega t)$$

Illustration 1. Consider the wave y = 5 sin (2x – 60t) mm. X is in centimeters and t is in second. Find (a) The Amplitude (b) The wave number (c) The wavelength (d) The frequency (e) The time period and (f) The wave velocity

Solution: Comparing the given equation with y = A sin (kx – ωt)

(a) Amplitude A= 5mm

(b) Wave number K=2 /cm

(c) Wave length $\lambda = 2\pi/2 = \pi$ cm

(d) Frequency $v = \frac{\omega}{2\pi} = \frac{60}{2\pi} = \frac{30}{\pi}$ Hz

(e) Time period $T = \frac{1}{v} = \frac{\pi}{30}$ sec

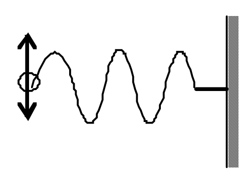
(f) Wave velocity v = $\nu\lambda$ = 30 cm.

Longitudinal and transverse wave:

In a longitudinal wave, the particles of the medium carrying the mechanical wave move back and forth along the direction of propagation. Sound in air is a longitudinal wave.

In a transverse wave, the particles of the medium oscillate in the direction perpendicular to the direction of propagation of wave, for example the waves in a taut string.

Sine wave travelling on a string



When one end of tight string is fixed and other end is displaced slightly up and down continuously, then it keeps on doing work on the string and the energy is continues to vibrate up and down from the left once the first disturbance has reached it. It receives the energy from left, transmits it to the right and process continues. The nature of vibration of any particle similar to that of the left end, the only difference being that the motion is repeated after a time delay of x/v.

When the left end vibrates i.e. x=0 in an SHM, The equation of motion of this end may then be

$$f(t) = A \sin\omega t$$

Where A= amplitude

ω = the angular frequency.

The wave produced by such a vibrating source is called a sine wave or sinusoidal wave.

Since the displacement of the particle at x = 0 is

Given by y= A sin ωt, the displacement of the particle at x at time t will be

$$y = f (t - x / v)$$

$$y = A \sin \omega (t - x / v)$$

The reason is that the wave moves along the string with a constant speed v and the displacement of the particle at x at time t was original as at x = 0 at time (t – x/ν).

Next, velocity

$$\frac{dy}{dt} = A\omega \cos\omega\,(t - x/v)$$

Here variable x is taken constant .It is the velocity of the same particle whose displacement should be considered as a function of time.

Energy and power of a traveling string wave:

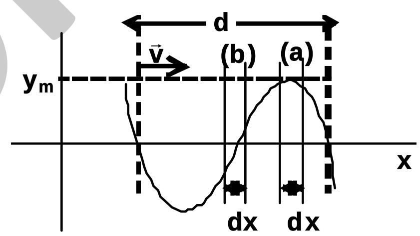
When a wave is set up on stretched string, the energy is provided for the motion of the string. As the wave moves away, it transports that energy as both kinetic energy and elastic potential energy

Kinetic Energy An element of the string of mass dm, oscillating transversely is SHM as the wave pass through it, has kinetic energy associated with its transverse velocit $y = \frac{\partial y}{\partial t}$. When the element is passing through its y = 0 position, its transverse velocity and thus kinetic energy is maximum. At the extreme position y = A, It transverse velocity and thus its kinetic energy is zero.

Elastic potential energy

To send a sinusoidal wave along the string, the wave must stretch the string. As a string element of length dx oscillates transversely, its length must increase and decrease in periodic way if the string element is to fit the sinusoidal wave form.

When the string element is at y = A position, its length has its normal undisturbed value dx, so its elastic potential energy is zero. At y = 0 position, elastic potential energy is maximum.



Energy of a plane progressive wave:

The kinetic energy dE associated with a string element of mass dm is given by

$$dE = \frac{1}{2}mv^2$$

Where v is the transverse speed of the oscillating string element.

$$v = \frac{\partial y}{\partial t} = \omega\, A\cos(\omega t - kx)$$

Thus $$dE = \frac{1}{2}(dm)\omega^2 A^2 \cos^2(\omega t - kx)$$

$$= \frac{1}{2}(\mu dx)\,\omega^2 A^2 \cos^2(\omega t - kx)$$

Rate of energy transmitted

$$\frac{dE}{dt} = \frac{1}{2}\mu\left(\frac{dx}{dt}\right)\omega^2 A^2 \cos^2(\omega t - kx)$$

$$= \frac{1}{2}\mu v\,\omega^2 A^2 \cos^2(\omega t - kx)$$

The averages rate at which K.E is transmitted is

$$\left(\frac{dE}{dt}\right)_{avg} = \frac{1}{4}\mu v\,\omega^2 A^2$$

($\because$ Average of $\cos^2(\omega t - kx)$ over integer multiples of wavelengths crossed is ½)

Elastic potential energy is also carried along with the wave, and at the same average rate.

The average power, which is the average rate at which energy of both kinds is transmitted by the wave,

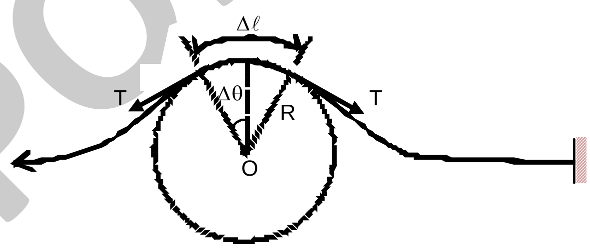
$$P_{avg} = 2\left(\frac{dE}{dt}\right)_{avg}$$

$$P_{avg} = \frac{1}{2}\mu v\,\omega^2 A^2$$

Wave Speed

The speed of any mechanical wave, transverse or longitudinal, depends on both an inertial property of the medium (to store kinetic energy) and an elastic property of the medium (to store potential energy).

Transverse wave in a stretched string



Consider a transverse pulse produced in a taut string of linear mass density μ. Consider a small segment of the pulse, of length Δl, forming an arc of a circle of radius R. A force equal in magnitude to the tension T pulls tangentially on this segment at each end.

Let us set an observer at the centre of the pulse which moves along with the pulse towards right. For the observer any small length Δl of the string as shown will appear to move backward with a velocity v.

Now the small mass of the string is in a circular path of radius R moving with speed v. Therefore the required centripetal force is provided by the only force acting, (neglecting gravity) is the component of tension along the radius.

The net restoring force on the element is

$$F = 2T\sin(\Delta\theta) \approx T(2\Delta\theta) = T\frac{\Delta l}{R}$$

The mass of the segment is $\Delta m = \mu\Delta$

The acceleration of this element toward the centre of the circle is

$$a = \frac{v^2}{R},$$ where v is the velocity of the pulse.

Using second law of motion,

$$T\frac{\Delta l}{R} = (\mu\Delta l)\left(\frac{v^2}{R}\right) \qquad \text{or} \qquad v = \sqrt{\frac{T}{\mu}}$$

Illustration 2. *The fundamental frequency of a sonometer wire increases by 5 hz if its tension increases by 21 %. How will the frequency be affected if its length is increased by 10 % ?*

Solution : $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$$f + 5 = \frac{1}{2l}\sqrt{\frac{1.21T}{\mu}}$$

by solving f = 50 Hz.

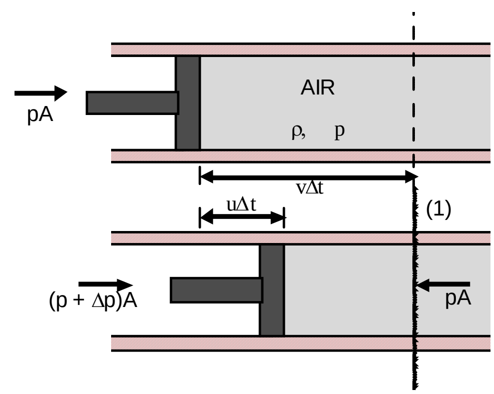
Next, $f' = \frac{1}{2(1.1)l}\sqrt{\frac{T}{\mu}} = \frac{f}{1.1}$ = 45.45 Hz.

Longitudinal wave in fluids

Sound wave in air is a longitudinal wave. As a sound wave passes through air, potential energy is associated with periodic compressions and expansions of small volume elements of the air. The property that determines the extent to which an element of the medium changes its volume as the pressure applied to it is increased or decreased is the bulk modulus B.

$$B = \frac{-\Delta p}{\Delta V/V}$$

Where $\frac{\Delta V}{V}$ is the fractional change in volume produced by a change in pressure Δp.



Suppose air of density ρ is filled inside a tube of cross-sectional area A under a pressure p. Initially the air is at rest.

At t = 0, the piston at the left end of the tube(as shown in the figure) is set in motion towards right with a speed u. After a time interval Δt, all portions of the air at the left of section 1 are moving with speed u whereas all portions at the right of the section are still at rest. The boundary between the moving and the stationary portions travels to the right with v, the speed of the elastic wave (or sound wave). In the time interval Δt, the piston has moved $u\Delta t$ and the elastic disturbance has traveled a distance $v\Delta t$.

The mass of air that has attained a velocity u in a time Δt is $\rho(v\Delta t)A$. Therefore, the momentum imparted is $[\rho v\Delta t\, A]u$. And, the net impulse acting is $(\Delta p\, A)\Delta t$

Thus, impulse = change in momentum

$$(\Delta p\, A)\Delta t = [\rho v(\Delta t)\, A]u$$

or $\Delta p = \rho v u$ . . . (1)

Since $B = \frac{\Delta p}{\Delta V/V}$

$\therefore$ $\Delta p = B\left(\frac{\Delta V}{V}\right)$

Where $V = Av\,\Delta t$ and $\Delta V = Au\,\Delta t$

$\therefore \qquad \frac{\Delta V}{V} = \frac{Au\Delta t}{Av\Delta t} = \frac{u}{v}$

Thus, $\quad \Delta p = B\frac{u}{v} \qquad \qquad ...(2)$

From (1) and (2) $\qquad v = \sqrt{\frac{B}{\rho}}$

Illustration 3. *Determine the speed of sound waves in water, and find the wavelength of a wave having a frequency of 242 Hz. Take B$_{water}$ = 2 ò 10^9 Pa.*

Solution: Speed of sound wave, $\quad v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2\times10^9}{10^3}} = 1414\,\text{m/s}$

Wavelength $\qquad \lambda = \frac{v}{f} = \frac{1414}{242} = 5.84\,\text{m}$

Speed of sound in an ideal gas

Newton assumed that the motion of sound waves in air is isothermal. In the case of ideal gas relation between pressure and volume during the isothermal process is

PV = constt.

By differentiating,

pdv + vdp = 0

$$\frac{dp}{dv} = -\frac{p}{v} \qquad ...(i)$$

Since $\qquad B = -v\frac{dp}{dv} \qquad ...(ii)$

From (i) and (ii)

$$B = -v\frac{dp}{dv} = -v \times -\frac{P}{v} = P \qquad ...(iii)$$

$$\therefore \qquad v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{P}{\rho}} \qquad ...(iv)$$

Laplace correction: Since the gaseous medium is a bad conductor of heat, the heat produced at a compression or rarefaction can not be conducted with surrounding medium therefore, taking relation in p and v for adiabatic process,

pV^γ = constant.

Where γ is the ratio of the heat capacity at constant pressure to that at constant volume.

After differentiating , we get

$$\frac{dp}{dV}V^\gamma + \gamma p V^{\gamma-1} = 0 \qquad \Rightarrow \qquad \frac{dp}{dV} = \frac{\gamma P}{v}$$

Since $\qquad B = -V\frac{dp}{dV} = \gamma P$

$$\therefore \qquad v = \sqrt{\frac{\gamma P}{\rho}}$$

Using the gas equation $\frac{P}{\rho} = \frac{RT}{M}$ where M is the molar mass.

Thus, $v = \sqrt{\frac{\gamma RT}{M}}$ (T = Temperature in kelvin)

Loudness

Human ear is sensitive for extremely large range of intensity. So a logarithmic rather than an arithmetic scale is convenient. Accordingly, intensity level β of a sound wave is defined by the equation.

Loudness, $\beta = 10 \log\left(\frac{I}{I_0}\right)$ decibel

$I_0 = 10^{-12}\ W/m^2$ is the reference intensity (threshold level for normal human ear) level to which any intensity I is compared.

Superposition of Waves

Two or more waves can traverse the same space independently of one another. Thus the displacement of any particle in the medium at any given time is simply the sum of displacements that the individual waves would give it. This process of the vector addition of the displacement of a particle is called superposition.

Interference

When two waves of the same frequency, superimpose each other, there occurs redistribution of energy in the medium which causes either a minimum intensity or maximum intensity which is more than the sum of the intensities of the individual sources. This phenomenon is called interference of waves. Let the two waves be

$$y_1 = A_1 \sin(kx - \omega t),\quad y_2 = A_2 \sin(kx - \omega t + \delta)$$

According to the principle of superposition

$$\begin{aligned} y &= y_1 + y_2 \\ &= A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \delta) \\ &= A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t)\cos\delta + A_2 \cos(kx - \omega t)\sin\delta \\ &= \sin(kx - \omega t)(A_1 + A_2\cos\delta) + \cos(kx - \omega t)(A_2 \sin\delta) \\ &= R \sin(kx - \omega t + \phi) \end{aligned}$$

where, $A_1 + A_2\cos\delta = R\cos\phi$ and $A_2 \sin\delta = R\sin\phi$

and $R^2 = (A_1 + A_2\cos\delta)^2 + (A_2\sin\delta)^2$

$= A_1^2 + A_2^2 + 2A_1A_2\cos\delta$

If I_1 and I_2 are intensities of the interfering waves and δ is the phase difference, then the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

Now, $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ for $\delta = 2n\pi$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ for $\delta = (2n + 1)\pi$

Standing waves

A standing wave is formed when two identical waves travelling in the opposite directions along the same line, interfere.

On the path of the stationary wave, there are points where the amplitude is zero, they are known as NODES. On the other hand there are points where the amplitude is maximum, they are known as ANTINODES.

The distance between two consecutive nodes or two consecutive anitnodes is $\frac{\lambda}{2}$.

The distance between a node and the next antinode is $\frac{\lambda}{4}$.

Consider two waves of the same frequency, speed and amplitude, which are travelling in opposite directions along a string. Two such waves may be represented by the equations

$y_1 = a \sin (kx - wt)$ and

$y_2 = a \sin (kx + wt)$

Hence the resultant may be written as

$y = y_1 + y_2 = a \sin (kx - wt) + a \sin (kx + wt)$

$y = 2 a \sin kx \cos wt$

This is the equation of a standing wave.

Note:

(i) In this equation, it is seen that a particle at any particular point 'x' executes simple harmonic motion and all particles vibrate with the same frequency.

(ii) The amplitude is not the same for different particles but varies with the location 'x' of the particle.

(iii) The points having maximum amplitudes are those for which 2a sin kx, has a maximum value of 2a, these are at the positions,

$kx = \pi/2, 3\pi/2, 5\pi/2, \ldots\ldots$

or $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots\ldots$

These points are called **antinodes.**

(iv) The amplitude has minimumn value of zero at positions where

$kx = \pi, 2\pi, 3\pi, \ldots\ldots$

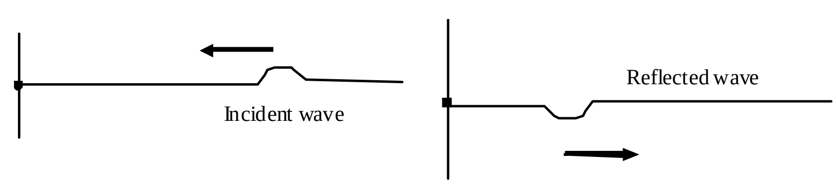
or $x = \lambda/2, \lambda, 3\lambda/2, 2\lambda \ldots\ldots$

These points are called **nodes.**

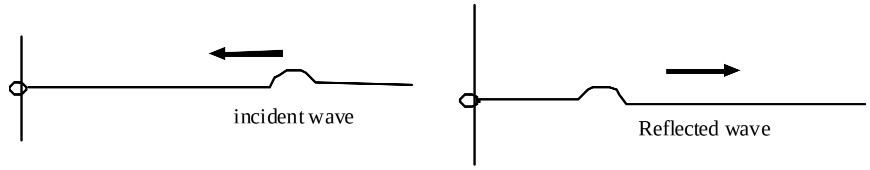
(v) Energy is not transported along the string to the right or to the left, because energy can not flow past the nodal points in the string which are permanently at rest.

Reflection of waves

(a) Waves on reflection from a fixed end undergoes a phase change of 180°.

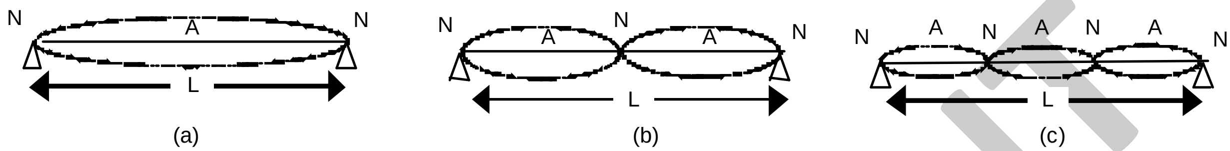


(b) While a wave reflected from a free end is reflected without a change in phase.



(c) In case of pressure wave there is no phase change when reflected from a denser medium or fixed end.

Stationary waves in strings



A string of length L is stretched between two points. When the string is set into vibrations, a transverse progressive wave begins to travel along the string. It is reflected at the other fixed end. The incident and the reflected waves interfere to produce a stationary transverse wave in which the ends are always nodes.

(a) In the simplest form, the string vibrates in one loop in which the ends are the nodes and the centre is the antinode. This mode of vibration is known as the fundamental mode and the frequency of vibration is known as the fundamental frequency or first harmonic.

Since the distance between consecutive nodes is $\lambda/2$

$$L = \frac{\lambda_1}{2} \qquad \therefore \qquad \lambda_1 = 2L$$

If f_1 is the fundamental frequency of vibration, then the velocity of transverse waves is given as ,

$$v = \lambda_1 f_1 \quad \text{or} \quad f_1 = v/2L \Rightarrow \quad v = 2Lf_1 \qquad \ldots (1)$$

(b) The same string under the same conditions may also vibrate in two loops, such that the centre is also the node.

$$\therefore \qquad L = 2\frac{\lambda_2}{2} \qquad \therefore \qquad \lambda_2 = L$$

If f_2 is the frequency of vibrations, then the velocity of transverse waves is given as,

$$v = \lambda_2 f_2 \qquad \therefore \quad v = L\,f_2 \quad \text{or } f_2 = v/L \quad \ldots (2)$$

The frequency f_2 is known as second harmonic or first overtone.

(c) The same string under the same conditions may also vibrate in three segments.

$$\therefore \qquad L = 3\frac{\lambda_3}{2} \qquad \therefore \qquad \lambda_3 = \frac{2}{3}L$$

If f_3 is the frequency in this mode of vibration, then,

$$v = \lambda_3 f_3 \qquad \therefore \qquad v = \frac{2}{3}L\,f_3 \quad \text{or } f_3 = 3v/2L \qquad (3)$$

The frequency f_3 is known as the third harmonic or second overtone. Thus a stretched string in addition to the fundamental mode, also vibrates with frequencies which are integral multiples of the fundamental frequencies. These frequencies are known as harmonics.

The velocity of transverse wave in a stretched string is given as

$$v = \sqrt{\frac{T}{\mu}}$$ where T = tension in the string.

μ = linear density or mass per unit length of string.

If the string fixed at two ends, vibrates in its fundamental mode, then

$$v = 2Lf \qquad \therefore \quad f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

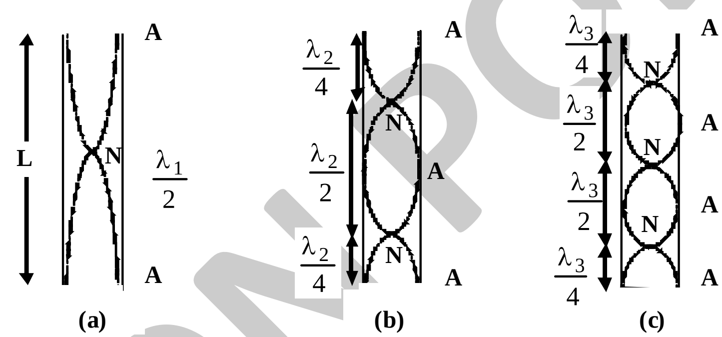
μ = volume of unit length $\times$ density

$= \pi r^2 \times 1 \times \rho = \pi \times \frac{D^2}{4} \times \rho$ where D = diameter of the wire, ρ = density.

Note : For a given fundamental frequency f, 2f is called upper octave of f and f/2 is called the lower octave of f.

Stationary waves in air column

Open pipe: If both ends of a pipe are open and a system of air is directed against an edge, standing longitudinal waves can be set up in the tube. The open end is a displacement antinode



(a) For fundamental mode of vibrations,

$L = \frac{\lambda_1}{2}$ $\therefore \lambda_1 = 2L$

$v = \lambda_1 f_1$ $\therefore$ $v = 2Lf_1$ ...(1)

(b) For the second harmonic or first overtone,

$L = \lambda_2$ or $\lambda_2 = L$

$v = \lambda_2 f_2$ $\therefore$ $v = Lf_2$ ...(2)

(c) For the third harmonic or second overtone,

$L = 3 \times \frac{\lambda_3}{2}$ $\therefore \lambda_3 = \frac{2}{3}L$

$v = \lambda_3 f_3$ $\therefore$ $v = \frac{2}{3}L\, f_3$

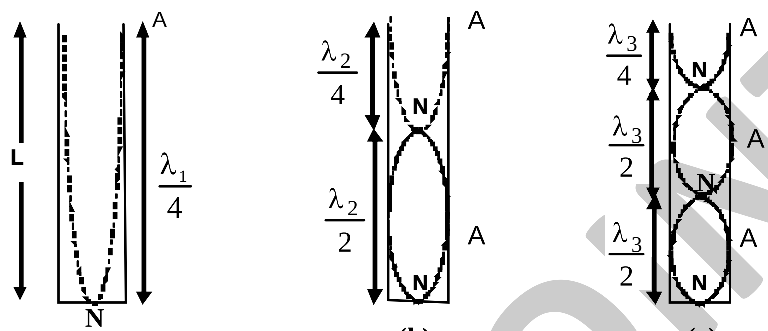
From (1) , (2) and (3) we get , $f_1 : f_2 : f_3 : .... = 1 : 2 : 3 : ....$

i.e. for a cylindrical tube, open at both ends, the harmonics excitable in the tube are all integral multiples of its fundamental.

In the general case, $\lambda = \frac{2L}{n}$ where n = 1,2 ......

Frequency $= \frac{v}{\lambda} = \frac{nv}{2L}$ where n = 1, 2.............

Closed pipe: If one end of a pipe is closed the reflected wave is 180° out of phase with the incoming wave. Thus the displacement of the small volume elements at the closed end must always be zero. Hence the closed end must be a displacement node.



(a) (b) (c)

(a) This represents the fundamental mode of vibration.

$$L = \frac{\lambda_1}{4} \qquad \therefore \ \lambda_1 = 4L$$

If f_1 is the fundamental frequency, then the velocity of sound waves is given as,

$v = \lambda_1 f_1 \ \therefore v = 4Lf_1$ . . . (1)

(b) This is the third harmonic or first overtone.

$$L = 3 \times \frac{\lambda_2}{4} \qquad \therefore \ \lambda_2 = \frac{4}{3}L$$

$v = \lambda_2 f_{-2} \qquad \therefore \ v = \frac{4}{3}L\, f_2$ . . . (2)

(c) This is the fifth harmonic or second overtone.

$$L = 5 \times \frac{\lambda_3}{4} \qquad \therefore \ \lambda_3 = \frac{4}{5}L$$

$v = \lambda_3 f_3 \qquad \therefore \ v = \frac{4}{5}L\, f_3$ . . . (3)

From (1) , (2) and (3) we get, $f_1 : f_2 : f_3 :$ .... = 1 : 3 : 5 : ....

In the general case, $\lambda = \frac{4l}{(2n+1)}$ where n = 0, 1, 2, ...

Velocity of sound = v

Frequency $= \frac{(2n+1)v}{4L}$ where n = 0, 1, 2.......

Beats: When two interfering waves have slightly different frequencies the resultant disturbance at any point due to the superposition periodically fluctuates causing waxing and waning in the resultant intensity. The waxing and waning in the resultant intensity of two superposed waves of slightly different frequency are known as beats.

Let the displacement produced at a point by one wave be

$$y_1 = A\sin(2\pi f_1 t - \phi_1)$$

and the displacement produced at the point produced by the other wave of equal amplitude as

$$y_2 = A\sin(2\pi f_2 t - \phi_2)$$

By the principle of superposition, the resultant displacement is

$$y = y_{1+}y_2 = A\sin(2\pi f_1 t - \phi_1) + A\sin(2\pi f_2 t - \phi_2)$$

$$y = 2A\sin\left\{2\pi\left(\frac{f_1+f_2}{2}\right)t - \left(\frac{\text{¤}_1-\text{¤}_2}{2}\right)\right\}\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t$$

$$y = R\sin\left\{2\pi\left(\frac{f_1+f_2}{2}\right)t - \left(\frac{\text{¤}_1-\text{¤}_2}{2}\right)\right\}$$

Where, $R = 2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t$

The time for one beat is the time between consecutive maxima or minima.

First maxima would occur when

$$\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t = +1$$

Then $2\pi\left(\frac{f_1-f_2}{2}\right)t = 0 \quad \therefore \quad t = 0$

For second maxima would occur when

$$\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t = -1$$

Then $2\pi\left(\frac{f_1-f_2}{2}\right)t = \pi$

or $t = \dfrac{1}{f_1-f_2}$

The time for one beat $= \left(\dfrac{1}{f_1-f_2} - 0\right) = \dfrac{1}{f_1-f_2}$

Similarly it may also be shown that time between two consecutive minima is $\dfrac{1}{f_1-f_2}$. Hence frequency of beat i.e. number of beats in one second

$$\text{Beat frequency} = f_1 \sim f_2$$

Doppler Effect

The apparent shift in frequency of the wave motion when the source of sound or light moves with respect to the observer, is called Doppler Effect.

Calculation of apparent frequency

Suppose v is the velocity of sound in air, v_s is the velocity of the source of sound(s), v_o is the velocity of the observer (O), and f is the frequency of the source.

(i) Source moves towards stationary observer.

If the source were stationary the f waves sent out in one second towards the observer O would occupy a distance v, and the wavelength would be v/f. If S moves with a velocity v_s towards O, the f waves sent out occupy a distance (v - v_s) because S has moved a distance v_s towards O in 1s. So the apparent wavelength would be

$$\lambda' = \frac{v - v_s}{f}$$

Thus, apparent frequency $f' = \dfrac{\text{Velocity of sound relative to O}}{\text{Wavelength of wave reaching O}}$

$$f' = \frac{V}{\lambda'} = f\left(\frac{v}{v - v_s}\right)$$

(ii) Source moves away from stationary observer.

Now, apparent wavelength

$$\lambda' = \frac{v + v_s}{f}$$

$\therefore$ Apparent frequency

$$f' = v/\lambda'$$

or $f' = f$

(iii) Observer moves towards stationary source.

$$f' = \frac{\text{Velocity of sound relative to O}}{\text{Wavelength of wave reaching O}}$$

here, velocity of sound relative to O = v + v_O

and wavelength of waves reaching O = v/f

$$\therefore \quad f' = \frac{v + v_0}{v/f} = f\left(\frac{v + v_0}{v}\right)$$

(iv) Observer moves away from the stationary source.

$$f' = \frac{v - v_0}{v/f} = f\left(\frac{v - v_0}{v}\right)$$

(v) Source and observer both moves toward each other.

$$f' = \frac{v + v_0}{\frac{v - v_s}{f}} = f\left(\frac{v + v_0}{v - v_s}\right)$$

(vi) Both moves away from each other.

$$f' = f\left(\frac{v - v_0}{v + v_s}\right)$$

(vii) Source moves towards observer but observer moves away from source

$$f = f'\left(\frac{v - v_0}{v - v_s}\right)$$

(viii) Source moves away from observer but observer moves towards source

$$f = f'\left(\frac{v + v_0}{v + v_s}\right)$$

Note : The velocities are with respect to the medium.

Illustration 3. *The siren of a police can emit a pure note at a frequency of 1125 Hz. Find the frequency that one can perceive in the vehicle under the following circumstances*

(a) one's car is at rest; police car moving towards listener at 29 m/s.

(b) Police car at rest, listener's car is moving towards police at 29 m/s.

(c) Both police and the listener's are moving towards each other at 14.5 m/s.

(d) Listener's car moving at 9 m/s white police car is chasing with a speed of 38 m/s.

Solution: (a) Here, v_0 = 0 m/s, v_s = 29 m/s

$$f' = f\left(\frac{v}{v - v_s}\right) = 1125 \times \left(\frac{343}{343 - 29}\right) = 1229\text{Hz}$$

(b) v_s = 0 m/s, v_0 = 29 m/s

$$f' = f\left(\frac{v + v_0}{v}\right) = 1125 \times \left(\frac{343 + 29}{343}\right) = 1220\text{Hz}$$

(c) v_0 = 14.5 m/s, v_s = 14.5 m/s.

$$f' = f\left(\frac{v + v_0}{v - v_s}\right) = 1125 \times \left(\frac{343 + 14.5}{343 - 14.5}\right) = 1224\text{Hz}$$

(d) v_0 = 9 m/s, v_s = 38 m/s

$$f' = f\left(\frac{v - v_0}{v - v_s}\right) = 1125 \times \left(\frac{343 - 9}{343 - 38}\right) = 1232\text{Hz}$$

SUMMARY

- Equation of Travelling Wave $y = f(x \pm vt)$
- A harmonic wave can be represented as

$$y = A\sin\frac{2\pi}{\lambda}(x \pm vt) \qquad \text{(Sinusoidal wave)}$$

or $\quad y = A\sin(kx \pm \omega t)$

The negative sign refers to the wave travelling along the positive x - axis, and versa.

$$k = \frac{2\pi}{\lambda} = \text{angular wave number}$$

$$\omega = \frac{2\pi}{T} = 2\pi f = \text{angular frequency}$$

$$v = \lambda f = \frac{\omega}{k} = \text{wave velocity}$$

- In sound waves there is a phase gap of $\pi/2$ between the displacement and pressure waves, i.e at displacement minima there is pressure maxima and vice versa.
- Transverse wave in a taut string

$$v = \sqrt{\frac{T}{\mu}} \qquad T = \text{tension}, \quad = \text{mass per unit length.}$$

- Longitudinal wave in a solid; $v = \sqrt{\frac{Y}{\rho}}$ $\quad$ Y = Young's Modulus , ρ= density

- Longitudinal wave in a fluid; $v = \sqrt{\frac{K}{\rho}}$ $\quad$ [K = Bulk Modulus]

- $\text{Intensity} = \frac{\text{Power}}{\text{unit area}} = 2\pi^2 f^2 A^2 \rho V$
- A standing wave is produced by the superposition of two identical waves travelling in opposite directions viz., $y_1 = a \sin [kx + \omega t]$ and $y_2 = a \sin (\omega t - kx)$, gives the standing wave, $y = 2a \sin \omega t \cos kx$
- The points having the maximum amplitude are those where 2a cos kx has a maximum value of 2a, these are at the position,

  $kx = 0, \pi, 2\pi, \ldots$

  i.e. $\quad x = 0, \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots\ldots\ldots$

  These points are called antinodes
- The amplitudes reaches a minimum value of zero at the positions

  where $kx = \pi/2, 3\pi/2, 5\pi/2, \ldots$

  or $\quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots\ldots\ldots$

  These points are called nodes.

- Energy is not transported along the string to the right or to the left, because energy cannot flow through the nodal points in the string which are permanently at rest.
- If two or more waves of slightly different frequencies are superimposed, the intensity of the resulting wave has alternate maxima and minima. The number of maximas in one second is called beat frequency.
  Beat frequency = $|f_1 - f_2|$
- Doppler Effect: The apparent shift in frequency of the wave motion when the source of sound moves with respect to the observer, is called **Doppler Effect**.

  Apparent frequency, $n' = \left(\dfrac{c + v_0}{c - v_s}\right) \times n$, where symbols represent their usual meaning.

FINAL EXERCISE

1. Derive the equation of a simple harmonic wave of angular frequency of (i) transverse (ii) longitudinal waves.
2. What are the essential properties of the medium for propagation of (i) transverse waves (ii) longitudinal waves.
3. Derive an expression for the intensity of the wave in terms of density of the medium, velocity of the wave, the amplitude of the wave and the frequency of the wave.
4. Write Newton's formula for the velocity of sound in a gas and explain Laplace's correction
5. What are beats? How are they formed? Explain graphically.
6. Discuss graphically the formation of stationary waves. Why are these wave called stationary waves? Define nodes and antinodes.
7. State three differences between stationary and travelling waves.
8. Derive the equation of a stationary wave and show that displacement nodes are pressure antinodes and displacement antinodes are pressure nodes?
9. What are the characteristics of musical sounds. Explain.
10. What is a decibel (symbol) db)? What is meant by 'threshold of hearing' and threshold of feeling'?
11. What is meant by quality of sound? Explain with examples?
12. Discuss the harmonics of organ pipes. Show that an open pipe is richer in harmonics.
13. Show that (i) the frequency of open organ pipes. is two times the frequency of the fundamental note of a closed pipe of same length (ii) to produce a fundamental note of same frequency, the length of the open pipe must be two times the length of the closed pipe.
14. Describe an experiment to demonstrate existence of nodes and antinodes in an organ pipes?
15. Explain Doppler's effect and derive an expression for apparent frequency. How does this equation get modified if the medium in which the sound travels is also moving.
16. Discuss the applications of Doppler's effect in (i) measuring the velocity of recession of stars, (ii) velocity of enemy plane by RADAR and (iii) velocity of enemy boat by SONAR?
17. Calculate the velocity of sound in a gas in which two waves of wavelengths 1.00m and 1.01m produce 10 beats in 3 seconds.

06 ELECTRIC CHARGE & ELECTRIC FIELD

ELECTRIC CHARGE

Electric charge, like mass, is one of the fundamental attributes of the particle of which the matter is made. Charge is the physical property of certain fundamental particles (like electron, proton) by virtue of which they interact with the other similar fundamental particles. To distinguish the nature of interaction, charges are divided into two parts (i) positive (ii) negative. Like charges repel and unlike charges attract.

SI unit of charge is coulomb and CGS unit is esu. $1\ C = 3 \times 10^{9}$ esu.

Magnitude of the smallest known charge is $e = 1.6 \times 10^{-19}$ C (charge of one electron or proton).

Charging of a body

Charging can be done by two methods;

1. Conduction   2. Induction

Charging by conduction

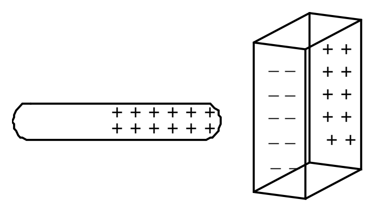
The process of charging from an already charged body can happen either by conduction or induction. Conduction from a charged body involves transfer of like charges. A positively charged body can create more bodies which are positively charged but the sum of the total charge on all positively charged bodies will be the same as the earlier sum.

Charging by induction

Induction is a process by which a charged body accomplishes creation of other charged bodies, without touching them or losing its own charge.

When a positively charged rod is brought near a plate, the free electrons are attracted by the +ve charge and move near to the rod.

Thus the portion nearer to rod becomes negatively charged and the portion farther from the rod becomes positively charged.



PROPERTIES OF ELECTRIC CHARGE

Quantization of charge

Charge exists in discrete packets rather than in continuous amount. That is charge on any body is the integral multiple of the charge of an electron

$\Rightarrow Q = \pm ne$, where n = 0, 1, 2, . . . where $e = 1.6 \times 10^{-19}$ C.

However, the step is so small that in most of the problems it can be taken as continuous variation.

Conservation of charge

Charge is conserved, i.e. total charge on an isolated system is constant. By isolated system, we mean here a system through the boundary of which no charge is allowed to escape or enter. This does not require that the amounts of positive and negative charges are separately conserved. Only their algebraic sum is conserved.

Within an isolated system, charges can be transferred from one part to another but the total charge is conserved.

Charges on a conductor

Static charges reside on the surface of the conductor.

COULOMB'S LAW:

Two point electric charges q_1 and q_2 at rest, separated by a distance r exert a force on each other whose magnitude is given by

$F = k\dfrac{q_1 q_2}{r^2}$ where k is a proportionality constant.

If between the two charges there is free space then

$k = \dfrac{1}{4\pi\varepsilon_0} = 9\times10^9 Nm^2C^{-2}$, Where ε_0 is the absolute electric permittivity of the free space.

and $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 / N-m^2

Illustration 1. *A polythene piece rubbed with wool is found to have negative charge of 3.2 ò 10^{J7} C.*

(a) Estimate the number of electrons transferred from wool to polythene.

(b) Is there a transfer of mass from wool to polythene? If yes, how much?

Solution: (a) Let n be the number of e^- getting transferred.

$\Rightarrow \quad n \times e = 3.2 \times 10^{-7}$

$\Rightarrow \quad 1.6 \times 10^{-19}\, n = 3.2 \times 10^{-7}$

$\Rightarrow \quad n = 2 \times 10^{12} \Rightarrow 2 \times 10^{12}$ electrons will get transferred.

(b) Mass transferred will be product of number of electrons and the mass of electron.

If m = mass getting transferred.

$\Rightarrow m = n \times (9.1 \times 10^{-31})$ kg

$m = 2 \times 10^{12} \times 9.1 \times 10^{-31}$

$m = 18.2 \times 10^{-31}$ kg

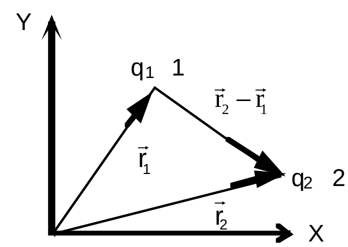
Illustration 2. Calculate force between two charges of 2 C each separated by 2 m in vacuum.

Solution: $F = \dfrac{kQ_1Q_2}{R^2} = \dfrac{9\times10^9\times2\times2}{2^2}$

$\Rightarrow F = 9 \times 10^9$ N

Coulombs law in Vector Relations

Suppose two charges q_1 and q_2 are placed at points 1 and 2. The position vectors are $\vec{r}_1$ and $\vec{r}_2$, $\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$. As per Coulombs law the force on q_2 applied by q_1 will be



$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}_{12}|^2}\hat{r}_{21}$$

where $\hat{r}_{21}$ is the unit vector in the direction 1 to 2.

Similarly, force on q_1 applied by q_2 is

which is equal in magnitude and opposite in direction to the vector $\vec{F}_{21}$

PRINCIPLE OF SUPERPOSITION

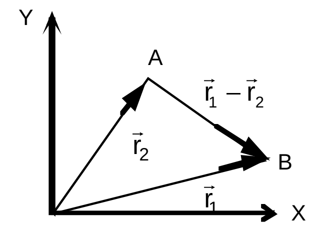
This principle tells us that if charge Q is placed in the vicinity of several charges q_1, q_2 ..... q_n, then the force on Q can be found out by calculating separately the forces $\vec{F}_1, \vec{F}_2 ... \vec{F}_n$, exerted by q_1, q_2, ... q_n respectively on Q and then adding these forces vectorially. Their resultant $\vec{F}$ is the total force on Q due to all of charges.

ELECTRIC FIELD

Electric field due to a point charge is the space surrounding it, within which electric force can be experienced by another charge.

Electric field strength or electric field intensity $(\vec{E})$ at a point is the electric force per unit charge experienced by a positive test charge at that point. Mathematically,

$\vec{E} = \frac{\vec{F}}{q_0}$, where q_0 is positive test charge.



In vector form, electric field at B due to charge Q at A, $\vec{E} = k\frac{q}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$

If electric field intensity is same at all points in the region, then the field is said to be uniform. Equispaced parallel lines represent uniform electric field. Arrow on the lines gives the direction of the electric field.

The electric field at a point due to several charges distributed in space is the vector sum of the fields due to individual charges at the point, $\vec{E} = \vec{E}_1 + \vec{E}_2 + ...\vec{E}_n$

The electric field due to continuous charge distribution at any point P, $\vec{E}_P = \frac{1}{4\pi\varepsilon_o}\int\frac{dq}{r^2}\hat{r}$

where dq is the charge on one element of the charge distribution and r is the distance from the element to the point under consideration and $\hat{r}$ is the unit vector directed from the position of elemental charge towards the point where electric field is to be found out.

Illustration 3. Two point charges 5 μC and –5 μC are located at 50 cm apart in vacuum.
(a) What is the electric field at the mid point O of the line AB joining the two charges.
(b) If a negative test charge of magnitude 2×10^{-9} C is placed at the above point, what is the force experienced by the test charge?

Solution: (a)

$$\vec{E}_0 = 2\times\frac{kq}{R^2} = \frac{2\times9\times10^9\times5\times10^{-6}}{(1/4)^2}\hat{j}$$

$$\vec{E}_0 = 16\times2\times9\times5\times10^3\,\frac{N}{C}\,\hat{j}$$

$$\vec{E}_0 = 144\times10^4\,\frac{N}{C}\hat{j}$$

(b)

$$\vec{F} = q\vec{E}_0 = 2\times10^{-9}\times144\times10^4(-\hat{i})N$$

$$\Rightarrow F = 288\times10^{-5}\left(-\hat{i}\right)N$$

LINES OF FORCE

It has been found quite convenient to visualize the electric field pattern in terms of lines of force. The electric field pattern vector at a point is related to imaginary lines of force in two ways. The line of force in an electric field is a curve such that the tangent at any point on it gives the direction of the resultant electric field strength at that point.

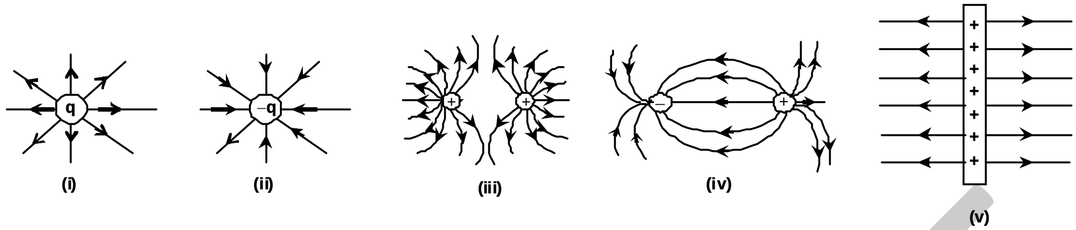
(i) Tangent to the lines of force at a point gives the direction of $\vec{E}$.

(ii) These lines of force are so drawn that their number per unit cross-sectional area in a region is proportional to the intensity of electric field.

(iii) Electric lines of force can never be closed loops.

(iv) Lines of force are imaginary.

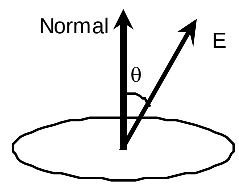
(v) They emerge from a positive charge and terminate on a negative charge.
(vi) Lines of force do not intersect.
Electric field lines produced by some typical charge distributions are given below.



The concept of field lines was invented by Faraday. The field lines are constructed purely to visualise the electric field. They have no physical existence.

Note : When a conductor has a net charge that is at rest, the charge resides entirely on the conductor's surface and the electric field is zero everywhere within the material of the conductor.

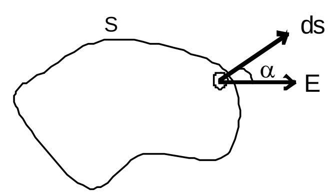
GAUSS'S LAW



The net "flow" of electric field through a closed surface depends on the net amount of electric charge contained within the surface. This " flow" is described in terms of the electric flux through a surface, which is the product of the surface area and the component of electric field perpendicular to the surface.

Flux is a scalar quantity and is added on per scalar addition rules. For non-uniform field and / or surfaces which are not plane,

$$¤ = \int \vec{E}.d\vec{s}$$



Direction of an area element is taken along its normal. Hence area can be treated as a vector quantity.

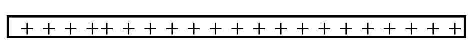
Gauss's law states that the total electric flux through a closed surface is proportional to the total electric charge enclosed within the surface. This law is useful in calculating fields caused by charge distributions that have various symmetry properties.

$$\text{Mathematically } \int \vec{E}.d\vec{s} = \frac{q}{£_0}$$

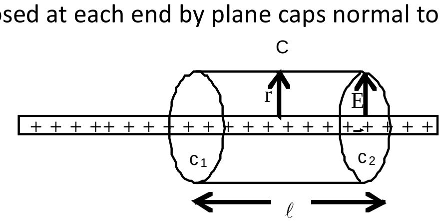
$\int \rightarrow$ means integral done over a closed surface.

Gauss's law can be used to evaluate Electric field if the charge distribution is so symmetric that by proper choice of a Gaussian surface we can easily evaluate the above integral.

Illustration 4. *Figure shows a section of an infinite rod of charge having linear charge density* ª *which is constant for all points on the line. Find electric field E at a distance r from the line.*



Solution: From symmetry, $\vec{E}$ due to a uniform linear charge can only be radially directed. As a Gaussian surface, we can choose a circular cylinder of radius r and length l, closed at each end by plane caps normal to the axis.



$$\varepsilon_0 \int \vec{E}.d\vec{s} = q_{in} \; ; \qquad \varepsilon_0\left[\int_c \vec{E}.d\vec{s} + \int_{c_1} \vec{E}.d\vec{s} + \int_{c_2} \vec{E}.d\vec{s}\right] = q_{in}$$

$$\varepsilon_0 E(2\pi r l) + \varepsilon_0 E.ds.\cos 90^\circ + \varepsilon_0 E.ds.\cos 90^\circ = \lambda l$$

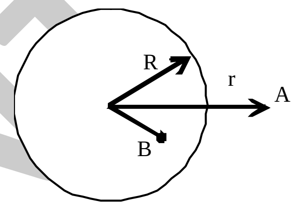
$$E = \frac{\lambda l}{\varepsilon_0 2\pi r l} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The direction of $\vec{E}$ is radially outward for a line of positive charge.

Illustration 5. Figure shows a spherical symmetric distribution of charge of radius R. Find electric field $\vec{E}$ for points A and B which are lying outside and inside the charge distribution respectively.

Solution: The spherically symmetric distribution of charge means that the charge density at any point depends only on the distance of the point from the centre and not on the direction. Secondly, the object can not be a conductor, or else the excess charge will reside on its surface.

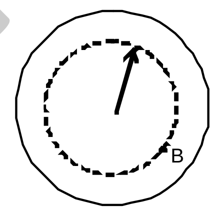
Now, apply Gauss's law to a spherical Gaussian surface of radius r



(r > R for point A), $\varepsilon_0 \int \vec{E}.d\vec{s} = q_{en}$

$\Rightarrow \quad \varepsilon_0 E\left(4\pi r^2\right) = q$

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$ where q is the total charge



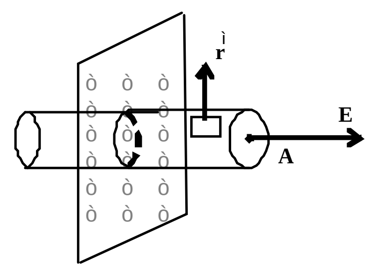
For point B (r < R), $\varepsilon_0 \int \vec{E}.d\vec{s} = \varepsilon_0 E\left(4\pi r^2\right) = q'$

$$q' = \frac{q\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = q\left(\frac{r}{R}\right)^3; \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q\left(\frac{r}{R}\right)^3}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$$

Illustration 6. *Determine the electric field due to a plane infinite sheet of charge.*

Solution: By symmetry, E will be perpendicular to the surface.

Gaussian surface is taken on a cylinder parallel to E.



$\int E.ds = 2\,A.E$

Charged enclosed = σ . A, where σ is charge density.

As per Gauss's theorem

$2\,A.E = \dfrac{\sigma A}{\varepsilon_0} \quad \therefore \quad E = \dfrac{\sigma}{2\varepsilon_0}$

The direction of the field is perpendicular to eth surface.

Illustration 7. *Determine the electric field near a large charged conducting surface ?*

Solution: Charged area being an equipotential surface, electric field will be perpendicular to the surface.

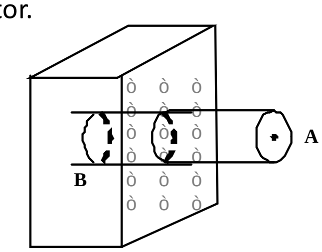
Taking a cylindrical Gaussian surface as shown in the figure,

Flux through area A = EΔS

Flux through cylindrical surface is zero as the area is perpendicular to the field.

Flux through area B is zero as there is no field inside the conductor.

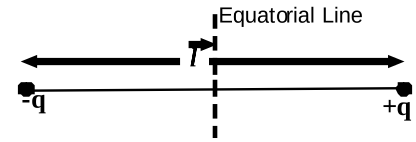
$\therefore \quad E\Delta S = \dfrac{q}{\varepsilon_0} = \dfrac{\pm\Delta S}{\varepsilon_0}$



$\Rightarrow \quad E = \dfrac{\pm}{\varepsilon_0}$

ELECTRIC DIPOLE

A set of two equal and opposite charges separated by a finite distance form an electric dipole. It is characterised by dipole moment vector $\vec{p}$.



(1) Charges (+q) and (–q) are called the poles of the dipole.

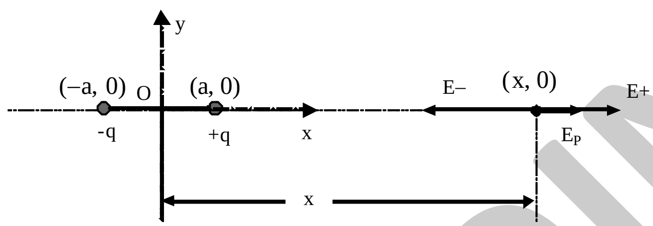
(2) $\vec{l}$ = the displacement vector from -ve charge to +ve charge.

(3) $\vec{p}$ = the dipole moment = $q\vec{l}$.

(4) The straight line joining the two poles (–q to q) is called axial line or the axis of the dipole.

(5) Perpendicular bisector of l is called equatorial line.

Electric field due to a dipole at axial point



Let the charges (–q) and +q are kept at point (–a, 0) & (a, 0) respectively in xy plane. The electric field at point P(x, 0) will be then;

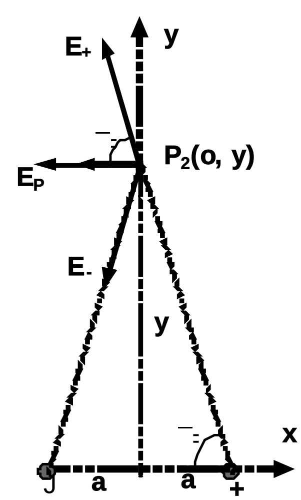
$$\vec{E}_{axial} = \vec{E}_{+q} + \vec{E}_{-q} = \frac{kq}{(x-a)^2}\hat{i} - \frac{kq}{(x+a)^2}\hat{i}$$, where $\hat{i}$ is the unit vector along axis.

$$= kq.\frac{\left[(x+a)^2 - (x-a)^2\right]}{\left(x^2-a^2\right)^2}\hat{i} = kq.\frac{(2x)(2a)}{\left(x^2-a^2\right)^2}\hat{i}$$

$$= \frac{2k\vec{p}x}{\left(x^2-a^2\right)^2}\hat{i}, \quad \text{as} \quad x >> a, \quad \vec{E} = \frac{2k\vec{p}}{x^3} \quad [p = 2aq]$$

$$\vec{E} = \frac{4aq}{4\pi\varepsilon_0 x^3}.\hat{i} = \frac{2\vec{p}}{4\pi\varepsilon_0 x^3}$$

Electric field on equatorial line



At P, $\vec{E}_+ = \frac{kq}{\left(y^2+a^2\right)}(-\cos\theta\hat{i} + \sin\theta\hat{j})$

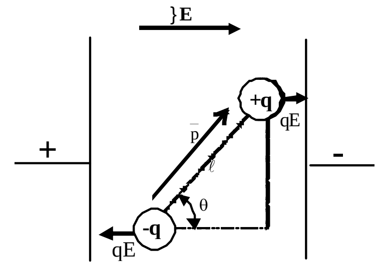
$$\vec{E}_- = \frac{kq}{\left(y^2+a^2\right)}(-\cos\theta\hat{i} - \sin\theta\hat{j})\ ; \quad \vec{E}_p = \vec{E}_+ + \vec{E}_- = \frac{-2kq}{\left(y^2+a^2\right)}\cos\theta\hat{i}$$

$$\Rightarrow \quad \vec{E}_P = -2\left[\frac{kq}{y^2+a^2}\right].\frac{a}{\left(a^2+y^2\right)^{1/2}}\hat{i} = \frac{-k(2aq)}{\left(y^2+a^2\right)^{3/2}}\hat{i} = \frac{-\vec{p}}{\left(y^2+a^2\right)^{3/2}}$$

For $a << y$, $E = \frac{kp}{y^3}$ or $\vec{E} = -\frac{k\vec{p}}{y^3} = -\frac{\vec{p}}{4\pi\varepsilon_0 y^3}$

Resultant $\vec{E}$ is directed oppositely to $\vec{p}$

Dipole in an external electric field



The net force on an electric dipole in a uniform external electric field is zero. However, the dipole in the presence of an external electric field experiences a torque and has tendency to align itself along the external electric field.

Torque on dipole = force x force arm

= qE (l sinθ) = (ql) (E sinθ)

= pE sin θ

or, $\vec{\tau} = \vec{p} \times \vec{E}$

The product of the charge q and separation l is the magnitude of a quantity called the electric dipole moment denoted by p.

The direction of $\vec{p}$ is along the dipole axis from the negative charge to the positive charge as shown in the figure.

As $\vec{E}$ is a conservative field, work done by an external agent in changing the orientation of the dipole is stored as potential energy in the system of a dipole present in an external electric field.

W = $\int \tau \, d\theta$

$\Rightarrow$ $\int pE \sin\theta \, d\theta = -pE\left[\cos\theta\right]_{\theta_1}^{\theta_2}$

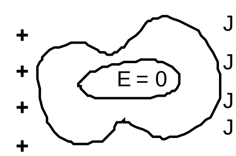
$\theta_2 = 0$, dipole is perpendicular to the field.

We assume, $\theta_1 = 90^o$ (as the datum for measuring potential energy can be chosen anywhere).

$\Rightarrow$ U = – pE cosθ or $U = -\vec{p}.\vec{E}$

Electrostatic shielding.

As the charges in a conductor reside on outer surface only and there is no field inside, any cavity inside a conductor is charge free and has no electric field even if the conductor is placed inside an external electric field. This fact is used in protecting sensitive instruments by providing electrostatic shielding by out side metal enclosure.



VAN-de-GRAAFF GENERATOR

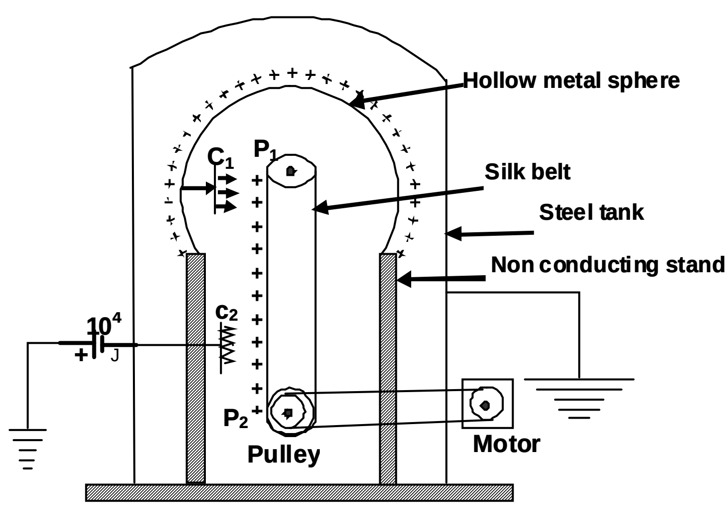
It is a device which is used to generate very high voltage in the order of 10^7 volts which is used to accelerate charged particle like electrons and protons for atomic experiments.

Working:

When charge is given to the hollow conductor from inner surface, the total charge is transferred to the outer surface of the conductor howsoever large its potential is.

Positive charge is provided by an external source to comb C_2. Charge is passed from comb C_2 to belt by ionization of air.

Belt is rotated with the help of motor so charge reaches near comb C_1. The charge density at sharp points of C_1 becomes high due to which action at a distance starts and belt is neutralized. The positive charge is transferred to the outer surface of the shell. As its charge increases, the potential also increases $\left(V = \dfrac{q}{4\pi\varepsilon_o R}\right)$. In this way, very high potential is generated on the metal shell.



When potential at the surface of metal sphere becomes very high, dielectric breakdown of the surrounding air take place due to which potential can not be increased further. This limit is proportional to the radius R(C = $4\pi\varepsilon_0$R) of the shell.

SUMMARY

- **Electric charge:** It is the property by which a particle electrically interacts with other particles. These can be positive or negative. Like charges repel each other and charges with opposite sign attract each other. Charges are quantized (minimum 1.6×10^{-19}C). Charge in an isolated system is conserved and the static charge resides on the surface of a conductor. Concentrations of charges is more on a smaller radius of curvature
- **Coulomb's law:** Electrostatic force between two point charges q_1 and q_2 separated by a distance r is,

  $$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

  Here $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 N - m^2/C^2$ and $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / m^2 - N$.

  It is the permittivity of free space.
- **Principle of Superposition**

  According to this principle, the net force acting on a charge q due to a number of charges q_1, q_2, ...q_n is equal to the vector sum of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 .... \vec{F}_n$, where $\vec{F}_1, \vec{F}_2, \vec{F}_3 .... \vec{F}_n$ are the forces on q due to q_1, q_2 ......q_n respectively.

  Thus $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 .... \vec{F}_n$

  Here, forces by charges are calculated separately without any effect of other charges.
- **Electric field**

  Electric field at $B(\vec{r}_2)$ due to a charge at $A(\vec{r}_1)$

  $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left|\vec{r}_2 - \vec{r}_1\right|^3}\left(\vec{r}_2 - \vec{r}_1\right),$$

  Electric field at a point is the force experienced by a unit positive charge kept at that point. Net field at a point is the vector sum of the fields due to individual charges.

  $$\vec{E} = \vec{E}_1 + \vec{E}_2 .... \vec{E}_n = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^3}\vec{r}$$ due to a continuous charge distribution.
- **Electric lines of force**

  The electric lines of force or field lines are imaginary and have no physical existence. Tangent at a point gives the direction of the net field at that point. Lines of force originate from positive charge and terminate on negative charge. They do not intersect each other. The number of lines of force per unit cross-sectional area placed perpendicular to the lines of force, is directly proportional to the intensity of electric field. Lines of force representing a uniform electric field are equi-spaced parallel straight lines.
- **Gauss' law**

  Gauss' law states that the total electric flux through a closed surface is proportional to the net charge inside the surface.

  $$\oint \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$
- **Force on a point charge in a Electric field**

  $$\vec{F} = q\vec{E}$$

- **Electric dipole**

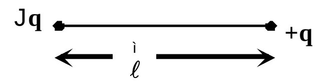
  It is a set of opposite charges separated by a small finite distance.

  Dipole moment $\vec{p} = q\vec{l}$

- **Electric field due to a dipole**

  (a) On the axis of dipole (at distance x)

  $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{x^3}$, axis of dipole is the line joining the charges and the parallel to $\vec{P}$



  (b) On an equatorial line

  $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3} \qquad \text{direction opposite to}$$

- **Torque on a dipole in an external field**

  $$\vec{\tau} = \vec{p} \times \vec{E}$$

  $$\Rightarrow \qquad \tau = pE\sin\theta$$

  P.E. of a dipole in electrostatic field

  $$U = -\vec{p}.\vec{E}$$

FINAL EXERCISE

1. Compare the electrical force with the gravitational force between two protons separated by a distance *x*. Take charge on proton as 1.60×10^{-19} C, mass of proton as 1.67×10^{-27} kg and Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

2. Four identical point charges +*q* each are placed at the four corners (one *q* at one corner) of a square of side 1. Find the force experienced by a test charge q_0 placed at the center of the square.

3. When are the electric field lines parallel to each other?

4. How many electrons should be removed from a metallic sphere to give it a positive charge = 6.4×10^{-7}C.

5. Consider an electric dipole of $q = 3.0 \times 10^{-6}$ C and $2l = 4 \times 10^{-10}$ m. Calculate the magnitude of dipole moment. Calculate electric field at a point $r = 6 \times 10^{-6}$ m on the equatorial plane.

6. A Charge $-q = 15 \times 10^{-6}$ C is placed on a metallic sphere of radius *R*=3.0 mm. Calculate the magnitude and direction of the electric field at a point *r* =15 cm from the center of the sphere. What will be the magnitude and direction of the field at the same point if 3.0 mm sphere is replaced by 9.0 mm sphere having the same Charge.

7. A charge of +15μC is located at the center of a sphere of radius 20 cm. Calculate the electric flux through the surface of the sphere.

8. A proton is placed in a uniform electric field $E = 8.0 \times 10^4$ NC^{-1}. Calculate the acceleration of the proton.

9. Two charges q_1 = 16μC and q_2 = 9 μC are separated by a distance 12m. Determine the magnitude of the force experienced by q_1 due to q_2 and also the direction of this force. What is the direction of the force experienced by q_2 due to q_1?

10. There are two identical metallic spheres *A* and *B*. *A* is given a charge +*Q*. Both spheres are then brought in contact and then separated. (i) Will there be any charge on *B* ? (ii) What will the magnitude of charge on *B,* if it gets charged when in contact with *A*.

11. A charged object has $q = 4.8 \times 10^{-16}$ C. How many units of fundamental charge are there on the object? (Take $e = 1.6 \times 10^{-19}$ C).

07 ELECTRIC POTENTIAL & CAPACITORS

ELECTRIC POTENTIAL

Potential at a point in an electric field is the amount of work done by an external agent against electric forces in moving a unit positive charge with constant speed from infinity to that point. Electric potential is a scalar quantity.

Potential due to a point charge

Taking the position of the point charge as origin, suppose point P, where the potential is to be ascertained is at a distance r.

Work done by the external agent = – work done by electric force.

Hence the required potential $V = -\int \vec{E}.\ \vec{d\ell} = -\dfrac{q}{4\pi \in_0} \int\limits_{\infty}^{r} \dfrac{dr}{r^2} = \dfrac{1}{4\pi \in_0} \dfrac{q}{r}$

The potential at a point due to a group of n point charges q_1 q_2, q_3 … q_n

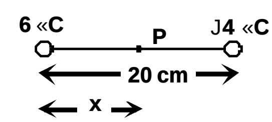
$V = V_1 + V_2 + \ldots + V_n$ (Scalar Sum)

$= \sum\limits_{i=1}^{n} \dfrac{1}{4\pi \in_0} \dfrac{q_i}{r_i}$

The electric potential due to a continuous charge distribution, $V = \int \dfrac{1}{4\pi \in_0} \dfrac{dq}{r}$

Illustration 1. *Two charges 6 C and J 4«C are placed on a line at a distance of 20 cm. Find out the position of point P where the potential is zero.*

Solution: Potential at point P = $\dfrac{6\times10^{-6}}{4\pi\varepsilon_0 x} + \dfrac{-4\times10^{-6}}{4\pi\varepsilon_0 (20-x)} = 0$



$\therefore$ 6(20 – x) = 4(x)

$\Rightarrow$ $x = \dfrac{120}{10} = 12\,\text{cm}$

Relation between field (E) and potential (V)

The negative rate of change of potential with distance along a given direction is equal to the component of the field along that direction.

i.e. $E_r = -\dfrac{dV}{dr}$

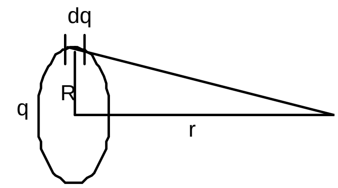
Illustration 2. *Kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100 V to a point at potential 200 V. Find the charge on the particle.*

Solution: q (200 – 100) = 10

$\Rightarrow q = \dfrac{10}{100} = 0.1\,C$

Potential due a uniformly charged ring

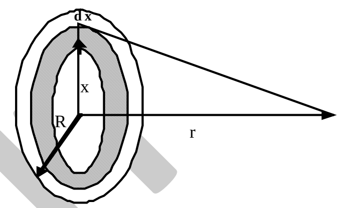
The element charge dq,



$$dv = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{r^2+R^2}}$$

$$v = \int dv = \frac{1}{4\pi\varepsilon_0\sqrt{r^2+R^2}}\int dq = \frac{q}{4\pi\varepsilon_0\sqrt{r^2+R^2}}$$

Potential due to a uniformly charged disc



If the charge on the disc = q, then $\pm = \frac{q}{\pi R^2}$

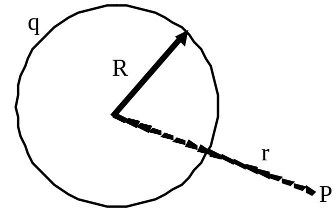
The elemental ring's area = $2\pi x dx$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\pm(2\pi x\,dx)}{\sqrt{r^2+x^2}}$$

$$V = \int dV = \frac{\pm}{2\in_0}\int_0^R \frac{x\,dx}{\sqrt{r^2+x^2}} = \frac{\pm}{2\in_0}\left[\sqrt{R^2+r^2}-r\right]$$

Potential due to a uniformly charged spherical shell

If the charge on the shell = q

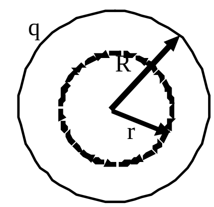


(i) For r > R, E = kq / r²

$$E = \frac{kq}{r^2}$$

Since $E = -\frac{dv}{dr}$; $v = -\int E dr.$

$$V = -\int_\infty^t \frac{kq}{r^2}dr = \frac{kq}{r}$$



(ii) For r = R, $V = \frac{kq}{R}$

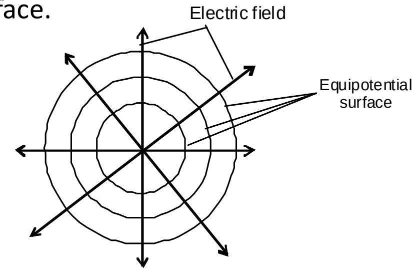
(iii) For r < R , E = 0

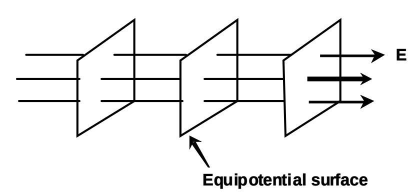
$$\Rightarrow \quad V = -\left[\int_\infty^R E\,dr + \int_R^t E\,dr\right] = -\int_\infty^R \frac{kq}{r^2}dr - \int_R^t 0\,dr = \frac{kq}{R}$$

Equipotential Surfaces

The locus of points of equal potential is called an equipotential surface. The electric field is perpendicular to the equipotential at each point of the surface.

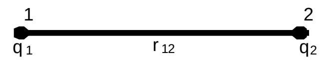
Examples of Equipotential surfaces





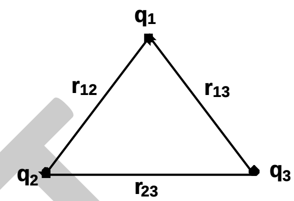
Electrostatic potential energy

The electric potential energy of a system of point charges is the amount of work done in bringing the charges from infinity in order to form the system. Two point charges q_1 and q_2 are separated at a distance r_{12}, Electric potential energy of the system q_1 and q_2



$$U = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r_{12}}$$

For three particle system q_1, q_2 and q_3

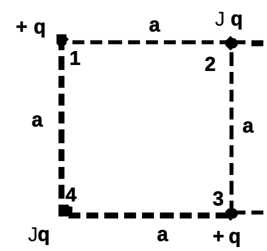


$$U = \frac{1}{4\pi \in_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

We can define electric potential at any point P in an electric field as,

$V_P = U_P/q$; where U_p, is the change in electric potential energy in bringing the test charge q from infinity to point P.

Illustration 3. *Determine the interaction energy of the point charges of the following setup.*

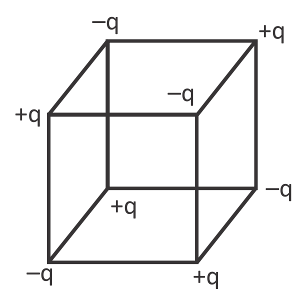


Solution : As you know the interaction energy of an assembly of charges is given by $\dfrac{1}{4\pi\varepsilon_0} \sum_{i \neq j}^{n} \dfrac{q_1 q_2}{r_{ij}}$

$\therefore$ $U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

$$= -\frac{kq^2}{a} + \frac{kq^2}{\sqrt{2}\,a} - \frac{kq^2}{a} - \frac{kq^2}{a} + \frac{kq^2}{\sqrt{2}\,a} - \frac{kq^2}{a} = -\frac{\sqrt{2}\,kq^2}{a}\left[2\sqrt{2} - 1\right] = \frac{q^2\left(4 - \sqrt{2}\right)}{4\pi\varepsilon_0 a}$$

Illustration 4. *Charges +q and Jq are located at the corners of a cube of side a as shown in the figure. Find the work done to separate the charges to infinite distance.*



Solution:

$$W_{external} = \Delta PE = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a} \left[-\frac{3}{1} + \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right] \times \frac{8}{2}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a} . \frac{4}{\sqrt{6}} \left[3\sqrt{3} - 3\sqrt{6} - \sqrt{2} \right]$$

Potential energy in an external field

Potential energy of a charge q placed in an external field E is $U = qV(r)$, where V(r) is the potential due to the field at point r.

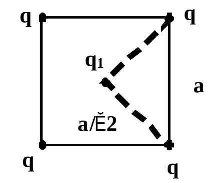
Here potential of charge q at P is considered as work done in bringing a unit positive charge from infinity to the point P.

For two charges q_1 and q_2 at point r_1 and r_2 in field E, the potential energy will be U = Work done in bringing the charge q_1 from infinity to point r_1 in the field E + Work done in bringing charge q_{-2} from infinity to point r_2 in field E + work done in bringing the charge q_2 from infinitely to r_2 against the field of charge q_1

$$U = q_1 (r_1) + q_2 V (r_2) \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

Illustration 5. *Four charges q are fixed at corners of a square of side a. Another charge q_1 is brought to the centre of the square. Find the potential energy of charge q_1*

Solution: Potential of four charges of kept at corners of the square at the centre of the square,

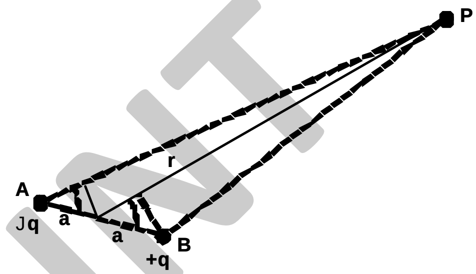


$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a/\sqrt{2}} \times 4 = \frac{\sqrt{2}\,q}{a\pi\varepsilon_0}$$

Work done in bringing the charge q, at this point (centre of square) against the field is

$$= qV = \frac{\sqrt{2}\,q_1 q_2}{\pi a \varepsilon_0}$$

Potential due to an electric dipole



$$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{PB} - \frac{q}{PA}\right]$$

When r >> a, PB = r – a cosθ

PA = r + a cosθ

$$V_P = \frac{1}{4\pi\varepsilon_0} \times \frac{2qa\cos\theta}{\left(r^2 - a^2\cos\theta\right)}$$

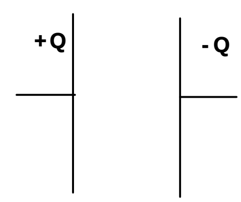
As r>>a, $a^2\cos^2\theta$ can be neglected in comparison to r^2.

$$\therefore \qquad V_P = \frac{1}{4\pi\varepsilon_0} . \frac{2qa\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

CAPACITANCE

If Q is the charge given to a conductor and V is the potential to which it is raised by this amount of charge, then it is found $Q \propto V$ or Q = CV, where C is a constant called capacitance of the conductor.

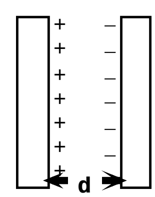
Capacitor



A pair of conductors separated by some insulating medium is called a capacitor. This medium is called dielectric of the capacitor. If Q units of the charge is given to one of the conductors, and thereby a potential difference V is set up between the conductors, the capacitance is then defined as

C = Q/V

Parallel Plate Capacitor



Parallel plate capacitor consists of two conductor plates kept at a small distance d. One plate is given a charge Q and the other one has a charge –Q. It is assumed that surface area A is much larger than separation d so that the effect of bending outward of electric field lines at the edges and the non-uniformity of σ (charge density) at the edges can be ignored.

Now taking field due to the surface charges, outside, on both sides of capacitor,

$$C = \frac{q}{v} = \frac{A\varepsilon_0}{d}$$

This result is valid for vacuum between the capacitor plates. For other medium, then capacitance will be $C = \frac{kA\varepsilon_0}{d}$, where k is the dielectric constant of the medium,

ε_0 = permittivity of free space = 8.85×10^{-12} $C^2/N\text{-}m^2$

Illustration 6. *A parallel plate capacitor is made of two square plats of 50 cm ò 50 cm size and separation between the plates of 0.5 mm. Calculate its capacitance.*

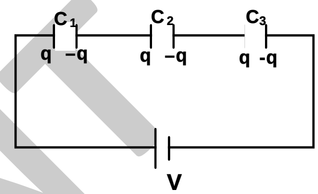
Solution: $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{8.85\times10^{-12}\times0.25}{0.5\times10^{-3}}$ = 4.425 10^{-9} F

C = 4.425 nF

That is the capacitance depends only on geometrical factors namely the plate area and plate separation.

Combination of Capacitors

Series Series combination



In series combination, each capacitor has equal charge for any value of capacitance, provided that capacitors are initially uncharged.

Charge q and –q will appear on first plate of the first capacitor and the last plate of the last capacitor, provided by the battery.

Then right plate of first capacitor will have charge –q and hence left plate of 2nd capacitor will have charge q and so on.

As shown in the figure, charge q & –q will appear on all capacitors connected in series.

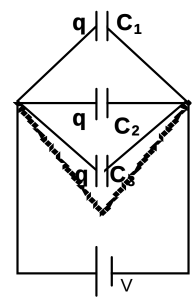
The total potential difference is the sum of potential differences on all capacitors

$$V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \qquad \frac{1}{C_{eff}} = \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

This is true for any number of capacitors in series.

Parallel combination



In parallel combination the potential differences of the capacitors connected in parallel are ... $_1$, q_2 and q_3 will appear on capacitors C_1, C_2 and C_3.

The total charge supplied by the battery, $Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$, where V is the potential difference across the capacitor and is same for all capacitors.

$$C_{eff} = \frac{Q}{V} = C_1 + C_2 + C_3$$

This can be generalised to parallel combination of any number of capacitors.

Sometimes it may not be easy to find the equivalent capacitance of a combination using the equations for series parallel combinations. For any combination one can proceed as follows:

Step - 1 : Connect an imaginary battery between the points across which the equivalent capacitance is to be calculated. Send a positive charge +Q from the positive terminal of the battery and –Q from the negative terminal of the battery.

Step - 2 : Write the charges appearing on each plates using charge conservation principle say, Q_1, Q_2 ..

Step - 3 : Assume the potential of the negative terminal of the battery be zero and that of positive terminal to be V, and write the potential of each of the plates say V_1, V_2 . . .

Step - 4 : Write the capacitor equation Q = CV for each capacitor. Eliminate Q_1, Q_2 . . . V_1, V_2 . . . .etc to obtain the equivalent capacitance $C = \dfrac{Q}{V}$.

Illustration 7. *Four parallel plate capacitances of 4«F, 5«F, 6 «F and 2«F are connected*
(a) in series (b) in parallel
Find equivalent capacitance.

Solution: (a) In series combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

$$\frac{1}{C} = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{2} = \frac{67}{60}$$

$$C_{eff} = \frac{60}{67}\mu F$$

(b) In parallel combination,

$$C_{eff} = C_1 + C_2 + C_3 + C_4 = (4 + 5 + 6 + 2)\ \mu C = 17\ \mu C$$

DIELECTRICS

When a dielectric is introduced between conductors of a capacitor, its capacitance increases. A dielectric is characterised by a constant 'K' called dielectric constant.

Dielectric constant

When a dielectric is placed in an external electric field, polarization occurs and it develops an electric field in opposition to the external one. As a consequence total field inside it decreases. If E be the total field inside the dielectric when it is placed in an external field E_0, then its dielectric constant K is given as $K = \frac{E_0}{E}$ (K > 1)

If a dielectric completely occupies the space between the conductors of a capacitor, its capacitance increases K times.

Hence in presence of a dielectric with dielectric constant K, the capacitance of a parallel plate capacitor $= \frac{K\varepsilon_0 A}{d} = KC_0$ where C_0 is the capacitance without dielectric.

Energy Stored in a Capacitor

Energy stored in a capacitor can be found out by calculating the work done while transferring total charge Q from one plate to another.

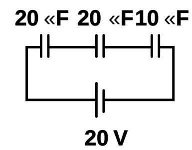
Suppose charge is being transferred from plate B to A. At a moment, charge on the plates are Q′ and –Q′. Then, to transfer a charge of dQ′ from B to A. the work done by an external force will be

$$dW = VdQ' = \frac{Q'}{C}dQ'$$

$$\text{Total work done } = \int_0^Q \frac{1}{C}Q'dQ' = \frac{Q^2}{2C}$$

$$\therefore \quad \text{Energy stored} = \frac{Q^3}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Illustration 8. *A battery of 20 V is connected to 3 capacitors in series as shown in the figure. Two capacitors are of 20«F each and one is of 10«F. Calculate the energy stored in the capacitors in the steady state.*



Solution: $$\frac{1}{C_{eff}} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{4}{20} = \frac{1}{5}$$

$C_{eff} = 5\mu F$

$\therefore$ Energy stored = $\frac{1}{2}CV^2 = \frac{1}{2} \times 5 \times 10^{-6} \times 20^2 = 10^{-3}$ J

Also, as $C = \varepsilon_o A/d$ and $V = E.d$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \left(\frac{1}{2}\epsilon_0 E^2\right)[Ad]$$

$\Rightarrow$ U (energy density) = Energy per unit volume = $\frac{1}{2}\epsilon_0 E^2$

If dielectric is introduced then, $U = \frac{1}{2}K\pounds_0 E^2$

This energy is stored in a capacitor in the electric field between its plates.

SUMMARY

- **Electric potential**

  Electric potential at a point in an electric field is the amount of work done by an external agent against electric force in moving a unit positive charge with constant speed from infinity to that point

  $$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \text{ (Potential of a point r distance from charge Q)}$$

- **Relation between potential and Field**

  $$E = -\frac{dV}{dx}$$

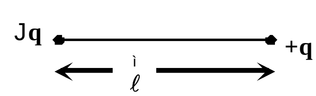
  Electric field is along the line of maximum rate of decrease of potential.
- **Equipotential Surface**

  Equipotential Surface is the locus of points of equal potential in an electric field. Electric field is always directed perpendicular to the equipotential surface.
- **Electrostatic potential energy:** It is the amount of work done in bringing the charges form infinity to form the system.

  $$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right) \text{ and so on.}$$

- **Electric dipole**

  It is a set of opposite charges separated by a small finite distance.

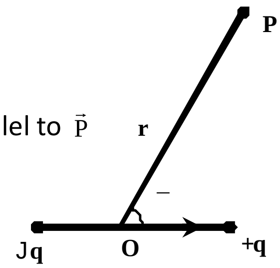
  

  Dipole moment $\vec{p} = q\vec{l}$

- **Electric field due to a dipole**

  (a) On the axis of dipole (at distance x)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{x^3}$$, axis of dipole is the line joining the charges and the parallel to $\vec{P}$

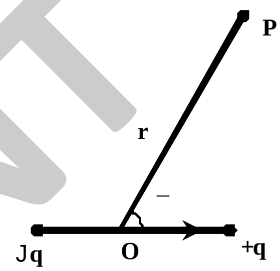


  (b) On an equatorial line

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$$ direction opposite to $\vec{P}$

  (c) On any other point

$$E = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$$



- **Potential due to a dipole**

$$V_P = -\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

- **Capacitance, capacitor**

  A capacitor consists of two metallic plates of area A at a distance d apart and opposite charges q and $-$q are given to the plates.

  Capacitance, $C = \frac{Q}{V}$

  For parallel plate capacitor $C = \frac{\varepsilon_0 A}{d}$

  For spherical capacitor $C = 4\pi\varepsilon_0 R$

Capacitors connected in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n}$$

Capacitors connected in parallel

$$C_{eq} = C_1 + C_2 + C_3 + . . . C_{-n}$$

Potential energy stored in a capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

Energy stored per unit volume

$$U = \frac{1}{2}\varepsilon_0 E^2$$

$$= \frac{1}{2}K\varepsilon E^2$$

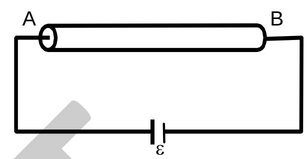
(with a dielectric of dielectric constant k introduced E is the net electric field in dielectric medium.

FINAL EXERCISE

1. Calculate the potential at a point P at a distance of 30 cm from a point charge q = 20μC
2. Three point charges q_1, q_2 and q_3, each of magnitude 200μC, are placed at the corners A, B and C respectively of an equilateral triangle. The length of the side is 10cm. Calculate the potential energy of the system.
3. The potential difference between the plates of a capacitor separated by 3mm is 12.0 V. Calculate the magnitude of E between the plates?
4. Two ions having charges $+e$ and $-e$ are 4.0×10^{-10} m apart. Calculate the potential energy of the system.
5. The plates A and B of a parallel plate capacitor have a potential difference of 15 V. A proton ($m = 1.67 \times 10^{-27}$ kg) is moved from the positive plate A to B. Calculate the speed of the proton near plate B.
6. Show that dimensionally the quantities Vq and $(½)mv^2$ are equivalent. The symbols carry the usual meaning.
7. Under what condition, the electric field between the plates of a parallel plate capacitor is uniform?
8. A metallic sphere of radius r has a charge $+q$. Calculate the work done in moving a test charge q_0 from one end of a diameter to its other end.
9. A parallel plate air capacitor of value C_0 is charged to a potential V_0 between the plates and $+q_0$ is charge on one plate. Separation between plates is d. A dielectric of dielectric constant $K = 3$ fills the space between the plates. Which of these quantities will change and why. (i) capacitance (ii) charge (iii) potential difference and (iv) field density?
10. A 3.0μF air capacitor is charged to a potential 12.0 V. A slab of dielectric constant $K = 7$ is made to fill the space. Calculate the ratio of the energies stored in the two systems.
11. A dipole of dipole moment $P = 3.5 \times 10^{-15}$ Cm is placed in a uniform electric field $E = 2.0 \times 10^4$ NC^{-1}. The dipole makes an angle of 60° with the field. Calculate the :
    (a) Potential energy of the dipole and
    (b) the torque on the dipole.
12. A metallic sphere of radius R has a charge $+q$ uniformly distributed on its surface. What is the potential at a point r (> R) from the centre of the sphere?
13. Calculate the work done when a point charge is moved in a circle of radius r around a point charge q.
14. The electric potential V is constant in a region. What can you say about the electric field **E** in this region ?
15. If electric field is zero at a point, will the electric potential be necessarily zero at that point.
16. Can two equipotential surfaces intersect?

08 ELECTRIC CURRENT

Flow of electric charge constitutes electric current. For a given conductor AB, if 'δQ' charge flows through a cross–section of area A in time 'δt', then the electric current through the conductor AB, is given as $I = \frac{\delta Q}{\delta t}$



The current so defined above, is the average current over the period δt. The instantaneous current is given as $I = \frac{dQ}{dt}$

Direction of electric current as defined above will be taken along the direction of flow of positive charge

Unit of Electric Current

The SI unit of electric current is ampere. It is denoted by A.

$$1\,\text{ampere}\,(A) = \frac{1\,\text{coulomb}\,(C)}{1\,\text{second}\,(s)} = 1\,\text{coulomb} / \text{second}$$

Current Density ($\vec{J}$)

To describe the flow of charge through a cross section of the conductor at a particular point, we use the term current density

$$J = \frac{i}{A}$$

where A is the total cross–sectional area of the surface. S.I. unit for current density is the ampere per square meter (A/m^2).

Illustration 1. ***How many electrons pass through a wire in 1 minute if the current passing through the wire is 200 mA ?***

Solution: $I = \frac{q}{t} = \frac{ne}{t}$

or, $n = \frac{It}{e}$

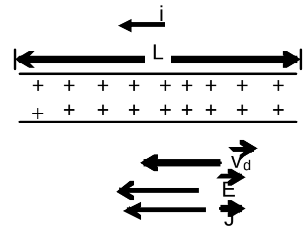
or, $n = \frac{200 \times 10^{-3} \times 60}{1.6 \times 10^{-19}} = 7.5 \times 10^{19}$

Illustration 2. ***One billion electrons pass through a conductor AB from end A to end B in 1 ms. What is the direction and magnitude of current?***

Solution : $i = \frac{Ne}{t} = \frac{(10^9)(1.6 \times 10^{-19}\,C)}{10^{-3}} = 1.6 \times 10^{-7}\,A$

$\Rightarrow$ i = 0.16 μA. The current flows from B to A.

Drift Speed



When no current flows through a conductor, its conduction electrons move randomly, with no net motion in any direction. When the conductor does have a current through it, these electrons still move randomly but now they tend to drift with a drift speed v_d in the direction opposite to that of the applied electric field which causes the current. The drift speed is small compared to the speed in the random motion of the electrons.

Let us assume that the positive charge carriers move with the same drift speed v_d across the wire's cross–sectional area A as shown in the figure.

The number of charge carriers in a length L of the wire is nAL, where n is the number of charge carriers per unit volume. The total charge of the carriers, each with charge e, in the length L is

$$q = (nAL)e$$

Since all charge carriers move along the wire with speed v_d, therefore total charge moves through any cross section of the wire in the time interval,

$$t = \frac{L}{v_d}$$

$\therefore$ Current $(i) = \dfrac{q}{t}$

or, $i = nAev_d$

or , $v_d = \dfrac{i}{nAe}$

or, $\vec{v}_d = \dfrac{i}{nAe} = \dfrac{\vec{J}}{ne}$

Illustration 3. ***What is the drift velocity of electrons in a copper conductor having a cross‑sectional area of 5 × 10^{-6} m² if the current is 10 A? Assume that there are 8 × 10^{28} electrons/m³.***

Solution: Given that,

$A = 5 \times 10^{-6}$ m², I = 10 A, and n = 8×10^{28} electrons / m³

Now, $v_d = \dfrac{1}{neA} = \dfrac{10}{8\times10^{28}\times16\times10^{-19}\times5\times10^{-6}}$ m/s

or, $v_d = 1.5625 \times 10^{-4}$ m/s.

Mobility

Mobility of a charge carrier is defined as the drift velocity of the charge carrier per unit electric field. It is generally denoted by μ. If v_d is drift velocity attained by free electrons on applying electric field E, then electron mobility is given by.

$$\mu = \frac{v_d}{E}$$

The SI unit of μ is $m^2\,V^{-1}s^{-1}$.

Substituting $v_d = \mu E$ in $I = neAv_d$. We get $I = neA\mu E$ this equation gives the relation between electron mobility and the current through conductor.

Relation between drift velocity and electric field

Due to random motion, the free electrons of metal collide with positive metal ions and undergo change in direction after every collision. So, the thermal velocities are randomly distributed in all possible directions. Therefore, the average velocity

$$\vec{u} = \frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_n}{N} = \text{zero}$$

Here, $\vec{u}_1, \vec{u}_2, ... \vec{u}_n$ are the individual thermal velocities of the free electrons at any given time and N is the total number of free electrons in the conductor.

However, when some potential difference V is applied across the two ends of a conductor of length ℓ, an electric field is set up. which is given by, $E = V / \ell$

Since charge on an electron is –e, each free electron in the conductor experiences a force $\vec{F} = e\vec{E}$ in a direction opposite to the direction of electric field.

If m is the mass of the electron, then acceleration produced is $\vec{a} = -\frac{e\vec{E}}{m}$

At any given time, an electron has a velocity such that $\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$, where $\vec{u}_1$ is the thermal velocity and $\vec{a}\tau_1$ is the velocity acquired by the electron under the influence of the applied electric field. where τ_1 being the time that has elapsed since the last collision.

Similarly, the velocities of the other electrons are $\vec{v}_2, \vec{v}_3 ..... \vec{v}_N$, such that $\vec{v}_2 = \vec{u}_2 + a\vec{\tau}_2$, $\vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3$, ..., $\vec{v}_n = \vec{u}_n + \vec{a}\tau_N$

The average velocity of all the free electrons in the conductor is equal to the drift velocity $\vec{v}_d$ of the free electrons. Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied electric field.

Now, $$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + ... + \vec{v}_N}{N}$$

or, $$\vec{v}_d = \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + ... + (\vec{u}_N + \vec{a}\tau_N)}{N}$$

or, $$\vec{v}_d = \frac{(\vec{u}_1 + \vec{u}_2 + ... \vec{u}_N)}{N} + \vec{a}\left(\frac{\tau_1 + \tau_2 + ... + \tau_N}{N}\right)$$ But, $\frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_N}{N} = 0$

$\therefore$ $$\vec{v}_d = \vec{a}\frac{\tau_1 + \tau_2 + ... + \tau_N}{N}$$ or, $\vec{v}_d = \vec{a}\tau$.

Here, τ is the average time elapsed between two successive collisions.

or, $$\vec{v}_d = -\frac{e\vec{E}}{m}\tau$$

OHM'S LAW

It states that current flowing between two points in a conductor is directly proportional to the potential difference between the two points.

i.e. $I \propto V$, provided the temperature is constant

$\Rightarrow$ $$\frac{V}{I} = R$$

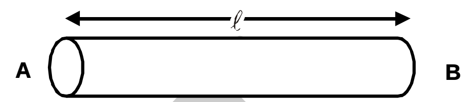
or, $V = IR$

where R is a constant.

The constant 'R' is called resistance of the conductor. Its value depends upon the nature of conductor, its dimensions and the surrounding (e.g. temperature).

Ohm's law is not universal (i.e. all conductors do not obey Ohm's law). Conductors obeying Ohm's law are called Ohmic conductors. However, resistance is always defined as the ratio V/I.

For a conductor of cross–sectional area A, resistance between the sections A and B separated by length ℓ is given by, $R_{AB} = \rho \frac{l}{A}$



where l = length of the conductor

A = Area of cross–section, and

ρ = resistivity or specific resistance of the conductor. (Its value depends upon the nature of the material of the conductor and its temperature.)

Unit of Resistance:

The SI unit of resistance is ohm. It is denoted by Ω. 1 ohm (Ω) = 1 volt ampere^{-1}

Conductance:

The reciprocal of resistance is called conductance. It is denoted by G.

$\therefore \quad G = \frac{1}{R}$; Its SI unit is ohm^{-1} or mho or siemen.

Unit of Resistivity:

We know that $R = \rho\ell/A$

$\therefore \quad \rho = RA/\ell$

In SI system, unit of resistivity = $\text{ohm} \times \frac{\text{metre}^2}{\text{metre}}$ = ohm–metre or $\Omega - m$

Illustration 4. A wire of resistivity '°' is stretched to double its length. What will be its new resistivity?

Solution: The resistivity of the wire does not depend on the length of the wire, so the resistivity will remain the same.

Illustration 5. Compare the resistances of two wires of same material. Their length are in the ratio 2 : 3 and their diameters are in the ratio 1 : 2.

Solution:

$$R_1 = \frac{\rho l_1}{A_2} = \frac{\rho\,(2x)}{\pi\,(y/2)^2} = \frac{8\rho x}{\pi y^2}$$

$$R_2 = \frac{\rho l_2}{A_2} = \frac{\rho\,(3x)}{\pi\,(2y/2)^2} = \frac{3\rho x}{\pi y^2}$$

$\Rightarrow R_1 : R_2 = 8 : 3$

Factors Affecting Electrical Resistivity

Let us consider a conductor of length ℓ and area of cross–section A. If n be the number of electrons per unit volume in the conductor and E is the applied electric field across the two ends of the conductor, then magnitude of drift velocity of electrons is

$$v_d = \frac{eE}{m}\tau \qquad \ldots (i)$$

The current flowing through the conductor due to drift of electrons is

$$I = nAv_d e \qquad \text{. . . (ii)}$$

From equations (i) and (ii)

$$I = \frac{nAe^2E\tau}{m} \qquad \text{. . . (iii)}$$

If V is the potential difference applied across the two ends of the conductor, then

$$E = \frac{V}{l} \qquad \text{. . . (iv)}$$

From equations (iii) and (iv)

$$I = \frac{nAe^2V\tau}{ml} \qquad \text{or,} \qquad \frac{V}{I} = \frac{ml}{ne^2\tau A}$$

or, $$R = \frac{m}{ne^2\tau}\frac{l}{A} \qquad \left[\because \quad R = \frac{V}{I}\right]$$

or, $$R = \rho\frac{l}{A}$$

It means the resistivity of the material of a conductor is

$$\rho = \frac{m}{ne^2\tau}$$

It shows that,

(i) $\rho \propto \frac{1}{n}$ [where n = number of free electrons per unit volume of the conductor]

(ii) $\rho \propto \frac{1}{\tau}$ [where τ = average relaxation time of free electrons in the conductor]

Variation of Resistivity with Temperature

ρ is independent of the shape and size of the conductor. It depends on temperature.

As temperature increases, ρ increases in case of Ohmic conductors.

At any temperature t, ρ is given by the following expression

$$\rho_{(t)} = \rho_0 (1 + \alpha\Delta T),$$

where ρ_0 = the resistivity at 0°C, and α = temperature coefficient of resistivity.

Also, $$\alpha = \frac{(\rho - \rho_0)}{\rho_0.\Delta T} \quad \Rightarrow \quad \alpha = \frac{1}{\rho}\frac{dp}{dT}$$

The resistivity of a semiconductor decreases rapidly with increasing temperature. We can explain these facts from the equation

$$\rho = \frac{m}{ne^2\tau} \qquad \text{. . . (i)}$$

(i) In case of conductors, the number of free electrons is fixed. Due to increase of temperature, the amplitude of vibration of atoms / ions increases. As a constant result of this, the collisions of electrons with the atoms become more effective and frequent. Therefore, τ decreases and hence ρ increases.

(ii) for semiconductors and insulators, resistivity increases with decreasing temperature.

Illustration 6. ***Calculate the resistance of a piece of a silver wire 0.50 m long and having diameter 2.74×10^{-4} m. Sp. resistance of silver ρ = 1.66×10^{-8} ohm metre.***

Solution: Given that, $\ell = 0.50$ m

$$r = \frac{2.74 \times 10^{-4}}{2} m = 1.37 \times 10^{-4} m$$

$$A = \pi r^2 = \pi(1.37 \times 10^{-4})^2 m^2 ; \qquad \rho = 1.66 \times 10^{-8} \text{ ohm m.}$$

We know that, $R = \rho\ell/A$

or, $$R = \frac{1.66 \times 10^{-8} \times 0.50 \times 7}{22 \times (1.37 \times 10^{-4})^2} = 0.1407\Omega$$

Illustration 7. ***Resistance of a conductor is 1.72 Ω at a temperature of 20°C. Find the resistance at 0°C and 100°C. Given the coefficient of resistivity is α = 0.00393/°C.***

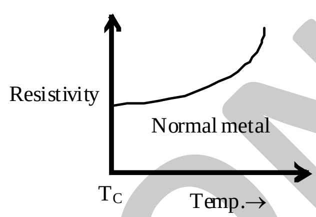
Solution: $R = R_0 (1 + \alpha \Delta T)$

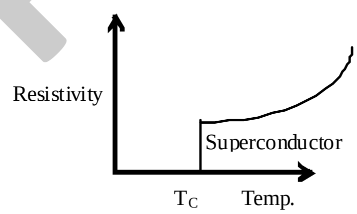
$R = 1.72\ \Omega\ [1 + 0.00393/°C\ (0°C - 20°C)] = 1.58\ \Omega$, and at T = 100°C

$R = 1.72\ \Omega\ [1 + (0.00393\ /°C]\ (100°C - 20°C)] = 2.26\ \Omega$

Superconductivity

The resistivity of certain materials suddenly becomes zero below a certain temperature. This phenomenon is called superconductivity and the material showing such behaviour is called superconductor. Above the critical temperature T_c (at which such transition occurs), the resistivity of the metal follows the trend of a normal metal as shown in graph A.





KIRCHHOFF'S LAW :

There are certain rules and techniques to solve the complicated circuits, containing resistances and batteries. These rules enable us to handle the complicated circuit systematically. The method given by Kirchoff's law is one of them. Kirchoff's law is incomplete without defining the basic terms.

Branch Point

The branch point in the network is the point where three or more conducts are joined.

Loop

A loop is any closed conducting path.

Junction rule

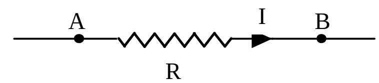
It is based on the law of conservation of charge. At a junction in a circuit, the total incoming current is equal to the total outgoing current. In other words, the algebraic sum of the currents at a junction is zero. A junction in a circuit is neither acts as a sink nor as a source of charge.

Loop rule

It is based on the law of conservation of energy. The algebraic sum of the potential drop around any closed path is zero.

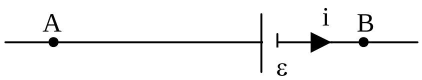
- In case of a resistor of resistance 'R' potential will decrease in the direction of current. Hence, for the shown conductor, potential drop across a resistance is IR

$$V_B - V_A = -IR$$



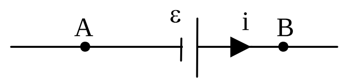
- For an emf source, the changes in potential will be obtained as illustrated below.

Emf = ε, internal resistance = r



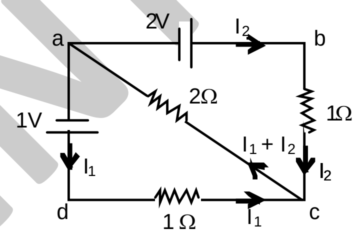
$v_B - v_A = -\varepsilon - ir$

Emf = ε, internal resistance = r



$v_B - v_A = \varepsilon - ir$

Illustration8. ***Calculate current in each resistance in the electrical circuit shown in the figure. The internal resistances of the cells are negligible.***



Solution: Applying Kirchhoff's second law to the mesh adca, we get

$-I_1 \times 1 - 2(I_1 + I_2) + 1 = 0$

$I_1 + 2(I_1 + I_2) = 1$

$3I_1 + 2I_2 = 1$ . . . (i)

Applying Kirchhoff's second law to the mesh abca, we get

$2 - 1 \times I_2 - 2(I_1 + I_2) = 0$

or, $I_2 + 2(I_1 + I_2) = 2$

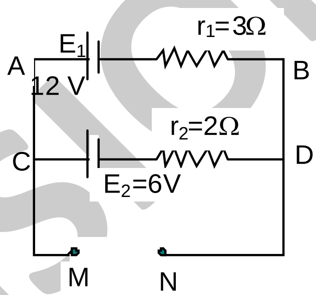
$2I_1 + 3I_2 = 1$ . . . (ii)

solving (i) and (ii), we get,

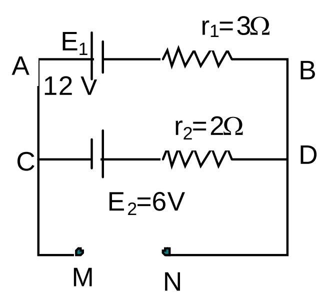
$I_1 = -0.2$ A and $I_2 = 0.8$ A

Current in 2Ω resistance = (0.8 – 0.2)A = 0.6 A.

Illustration 9. ***What is the potential difference between the points M and N for the circuits shown in the figures, For case I and case II?***

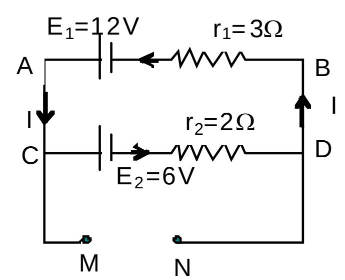


Case I



Case II

Solution : **Case I:**



Case I

$$I = \frac{E_1 - E_2}{r_2 + r_1} = \frac{12 - 6}{3 + 2} = 1.2 \text{ A}$$

So for cell E_1, $v_A - E_1 + Ir_1 = v_B$

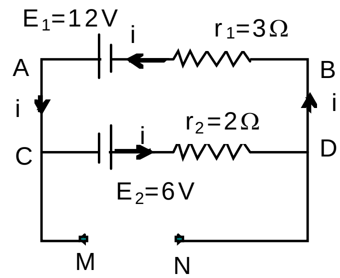
i.e. $v_A - v_B = E_1 - Ir_1$

$= 12 - 1.2 \times 3 = 8.4$ V

For cell E_2, $v_C - E_2 - Ir_2 = v_D$

i.e. $v_C - v_D = 6 + 1.2 \times 2 = 8.4$ V

Hence, $v_C - v_D = v_A - v_B = v_M - v_N = 8.4$ V



CaseI II

Case II:

$$I = \frac{E_1 + E_2}{r_1 + r_2} = \frac{12+6}{3+2} = 3.6A$$

For cell E_1,

$$v_A - E_1 + Ir_1 = v_B \text{ i.e. } v_A - v_B = E_1 - Ir_1$$
$$= 12 - 3.6 \times 3 = 1.2 \text{ V}$$

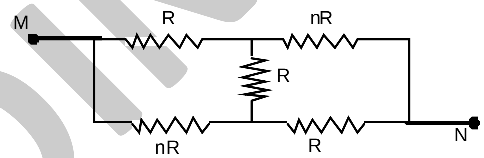
For cell E_2,

$$v_c + E_2 - Ir_2 = v_D$$

i.e. $v_C - v_D = -E_2 + Ir_2 = -6 + 3.6 \times 2 = 1.2$ V

Hence, $v_A - v_B = v_C - v_D = v_M - v_N = 1.2$ V

Illustration 10. ***Find the equivalent resistance between M and N.***



Solution : **For loop 1:**

$$-I_1R + R(I - 2I_1) + nR(I - I_1) = 0$$

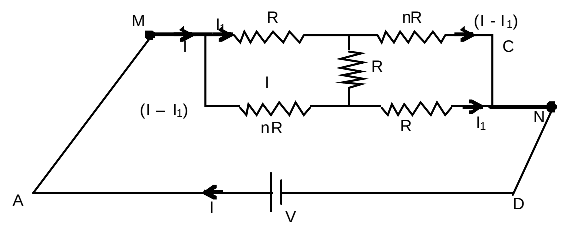
$$\Rightarrow I_1 (R + 2R + nR) = I (R + nR)$$

$$\Rightarrow I_1 = \frac{I(n+1)}{(n+3)}$$

For loop AMCNDA ; we have from Kirchoff's loop law

$$-I_1R - (I - I_1)nR + V = 0$$

$$\Rightarrow I_1 (R - nR) = V - nIR$$



$$\Rightarrow I_1 = \frac{I(n+1)}{(n+3)} R(1-n) + nIR = V$$

$$\Rightarrow IR\left[\frac{1-n^2}{n+3} + n\right] = V$$

$$\Rightarrow V = IR\left(\frac{1+3n}{n+3}\right)$$

$$\therefore R_{eq} = \left(\frac{3n+1}{n+3}\right)R$$

Carbon Resistors

To make a carbon resistor, carbon with a suitable binding agent is modulated into a cylinder. Wire leads are attached to this cylinder. The resistor is enclosed in a plastic or ceramic jacket. The resistor is connected to the circuit by means of two leads. Carbon resistors of different values are commercially available. They are widely used in electronic circuits of radio, amplifier, etc.

Colour Code for Carbon Resistors

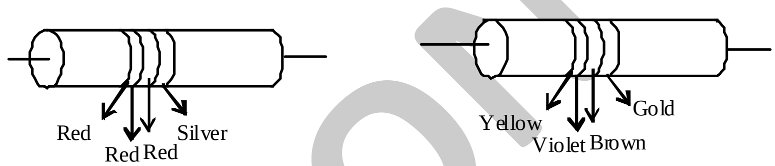
The resistances of a carbon resistor are indicated by means of colour code printed on it. The resistor has a set of coJaxial coloured rings on it with their significance as indicated in the table shown.

Colour	Number	Multiplier	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			**20**

To read the value of a carbon resistance, the following sentence serves as an aid to memory.

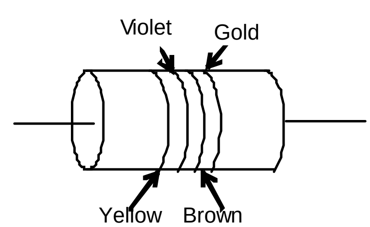
BBROY Great Britain very good wife

From the left end, first two bands indicate the first two significant figures of the resistance in ohms. The third band indicates the decimal multiplier and the last band indicates the tolerance or possible variation in percent about the indicated value. In the absence of fourth band, tolerance is $\pm$ 20%.



(a) (b)

Illustration 11. **The *figure shows a colour–coded resistor. What is the resistance of this resistor?***

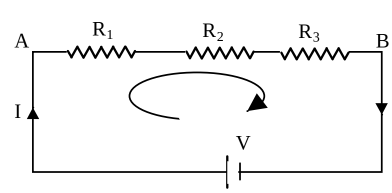


Solution: The number for yellow colour is 4. The number for violet colour is 7. Brown colour gives a multiplier of 10^1. Gold indicates a tolerance of 5%. So, the resistance of the given resistor is $47 \times 10\ \Omega \pm$ 5%.

GROUPING OF RESISTANCES

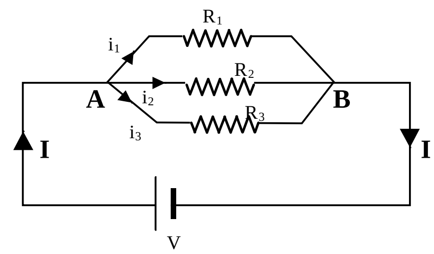
Series Combinations

Let the equivalent resistance between A and B equals R_{eq}, By definition,



$$R_{eq} = \frac{V}{I} \qquad \ldots (1)$$

Using Kirchoff's 2nd rule for the loop shown in figure,



$V = IR_1 + IR_2 + IR_3$ . . . (2)

From (1) and (2), $R_{eq} = R_1 + R_2 + R_3$

Parallel Combinations

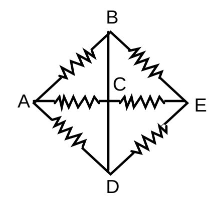
Here again, $R_{eq} = \dfrac{V}{I}$ . . .(1)

$I = i_1 + i_2 + i_3 = \dfrac{V}{R_1} + \dfrac{V}{R_2} + \dfrac{V}{R_3}$ . . . (2)

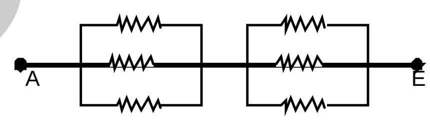
From (1) and (2)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Illustration 12. Calculate the equivalent resistance between points A and E as shown in the figure. Each resistance is of 2 t .



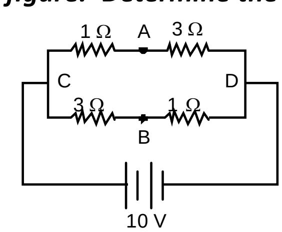
Solution: The points B, C and D are at the same potential. So, resistances AB, AC and AD are in parallel. Similarly, the resistances BE, CE and DE are in parallel. So, an equivalent of the given network is as under.



Parallel combination of 2 Ω, 2 Ω and 2 Ω gives $\dfrac{2}{3}\Omega$.

$\therefore$ $R_{AE} = 2 \times \dfrac{2}{3}\Omega = \dfrac{4}{3}\Omega = 1.33\Omega$

Illustration 13. A battery of emf 10 V is connected to resistances as shown in the figure. Determine the potential difference between A and B.



Solution: Total resistance = $\dfrac{4 \times 4}{4 + 4} = 2\Omega$

Current I = $\dfrac{10V}{2\Omega} = 5A$

Since the resistances of both the branches are equal, therefore the current of 5 A shall be equally distributed.

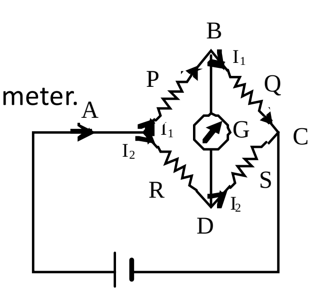
Current through each branch = $\dfrac{5}{2}A = 2.5A$

$V_c - V_A = 2.5 \times 1 = 2.5$ V

$V_c - V_B = 2.5 \times 3 = 7.5$ V.

$V_A - V_B = (V_c - V_B) - (V_c - V_A) = 7.5 - 2.5 = 5.0$ V

Wheatstone Bridge



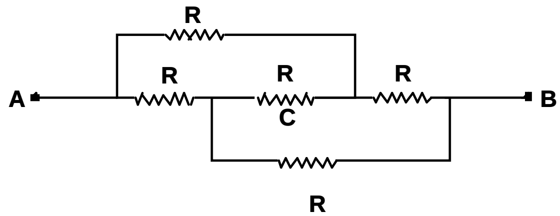
For a certain adjustment of Q, $V_{BD} = 0$, then no current flows through the galvanometer.

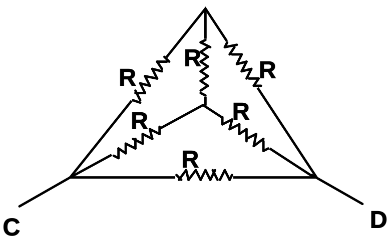
$\Rightarrow$ $V_B = V_D$ or $V_{AB} = V_{AD} \Rightarrow I_1.P = I_2.R$

Likewise, $V_{BC} = V_{DC} \Rightarrow I_1.Q = I_2.S$

Dividing, we get, $\dfrac{P}{Q} = \dfrac{R}{S}$

Illustration 14. What's the effective resistance of following circuits?

(a) 

(b) 

Solution: (a) It is a Wheatstone bridge that is balanced. Hence, the central resistance labeled 'C' can be assumed as ineffective.

$\Rightarrow \quad R_{eq} = R.$

(b) The resistor R is in parallel with a balanced Wheatstone bridge.

$$\Rightarrow \quad R_{eq} = \frac{R.R}{R+R} = \frac{R}{2}$$

MEASURING INSTRUMENTS

Galvanometer:

It is used to detect very small current. It has negligible resistance.

Ammeter

It is an instrument used to measure current. It is put in series with the branch in which current is to be measured. An ideal Ammeter has zero resistance. A galvanometer with resistance G and current rating i_g can be converted into an ammeter of rating I by connecting a suitable resistance S in parallel to it.

Thus, $S(i - i_g) = i_g G$

$$\Rightarrow \quad S = \frac{i_g G}{i - i_g}$$

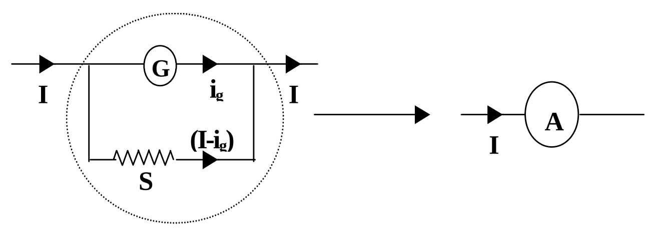


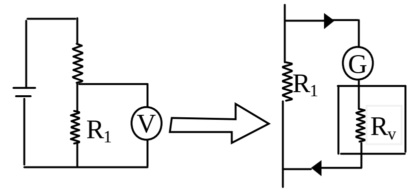
Illustration 15. A galvanometer having a coil resistance of 100 t gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into ammeter giving full scale deflection for a current of 10 A?

Solution:

$$S = \frac{i_g.G}{i - i_g} = \frac{(10^3 A)(100\Omega)}{(10 - 10^{-3})A} = \frac{0.1}{9.99}$$

$$S = \frac{1}{99.99}\Omega \approx 10^{-2}\Omega$$

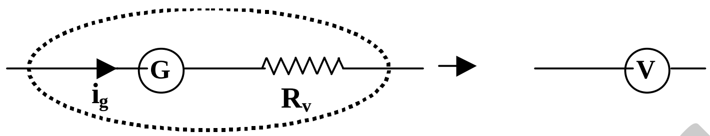
Voltmeter



It is an instrument to find the potential difference across two points in a circuit.

It is essential that the resistance R_v of a voltmeter be very large compared to the resistance of any circuit element with which the voltmeter is connected. Otherwise, the metre itself becomes an important circuit element and alters the potential difference that is measured.

For an ideal voltmeter $R_V = \infty$.



$i_g(G + R) = V \qquad \Rightarrow \qquad R = \frac{V}{i_g} - G$

Illustration 16. ***A galvanometer having a coil resistance of 100 t gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V?***

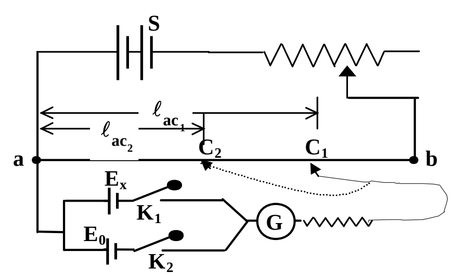
Solution: $V = I_g [G + R_v]$

$10 = (10^{-3})\,(100 + R_v)$

$$\Rightarrow \qquad R_v = \left(\frac{10}{10^{-3}}\right) - 100 = 9{,}900\Omega = 9.9\,k\Omega$$

POTENTIOMETER

A potentiometer is an instrument that measures the terminal potential difference with high accuracy without drawing any current from the unknown source. It is based on the principle that if constant current is passed through a wire of uniform cross–section, then potential difference across any segment of the wire is proportional to its length.



The given diagram shows a typical arrangement to measure emf E_x of a battery.

The wire ab is of uniform cross–section and carries a constant current supplied by battery S. First the switch K_1, is closed and K_2 is kept open. The slider is moved on the wire ab till we get zero deflection in the galvanometer. If C_1 is the corresponding point in the wire,

$$E_x = V_{ac_1}$$

Now, the experiment is repeated with key K_1 open and K_2 closed. This time, if the null deflection is obtained with contact on wire at C_2,

$$E_0 = V_{ac_2} \quad (E_o \text{ is known})$$

Now, $\dfrac{E_x}{E_0} = \dfrac{V_{ac_1}}{V_{ac_2}} = \dfrac{l_{ac_1}}{l_{ac_2}}$, where l_{ac_1} and l_{ac_2} are the lengths of segments ac_1 and ac_2 respectively.

Illustration17. ***A 10 m long wire of uniform cross – section and 20 Ω resistance is fitted in a potentiometer. The wire is connected in series with battery of 5 volt, alongwith an external resistance of 480 Ω If an unknown emf E is balanced at 6.0 m length of this wire, calculate (i) the potential gradient of the potentiometer wire, (ii) the value of the unknown emf E.***

Solution: $I = \dfrac{5}{480+20} A = 0.01A$

P.D. across the potentiometer wire = (0.01 $\times$ 20) V = 0.2 V

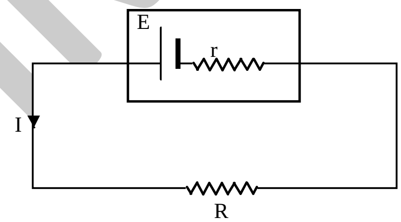
(i) Potential gradient = $\dfrac{0.2V}{10m}$ = 0.02 Vm^{-1}

(ii) E = (0.02 $\times$ 6.0) V = 0.12 volt.

ENERGY, POWER AND HEATING EFFECT OF THE CURRENT

When a constant current I flows for time t from a source of emf E, then the amount of charge that flows in time t is Q = It. Electrical energy delivered W = Q.V = VIt

Thus, power given to the circuit = W/t = VI or V^2/R or I^2R



In the circuit, we can write, E. I = $I^2R + I^2 r$,

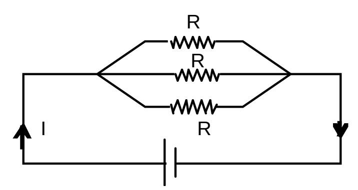
where EI is the rate at which chemical energy is converted to electrical energy, I^2R is power supplied to the external resistance R and I^2r is the power dissipated in the internal resistance of the battery.

An electrical current flowing through, conductor produces heat in it. This is known as Joule's effect. The heat developed in Joules is given by H = I^2.R.t, where I = current in Ampere, and R = resistance in ohms,

t = time in seconds.

The equation H = I^2Rt is also known as Joule's law of heating

Illustration 18. ***Three equal resistances, each of R Ω, are connected as shown in figure. A battery of emf 2V and internal resistance 0.1 Ω is connected across the circuit. Calculate the value of R for which the heat generated in the circuit is maximum?***



Solution: The given network is a parallel combination of three resistances.

Combined resistance $R' = \dfrac{R}{3}$

Current (I) = $\dfrac{E}{R/3+r}$

Power (P) = $\left(\dfrac{E}{R/3+r}\right)^2 \dfrac{R}{3} = \dfrac{E^2 R/3}{\left[\dfrac{R}{3}-r\right]^2 + \dfrac{4Rr}{3}}$

For maximum power, $\dfrac{R}{3} - r = 0$

or R = 3r = 0.3 Ω

Illustration 19. ***An electric bulb rated 220 V and 60 W is connected in series with another electric bulb rated 220 V and 40 W. The combination is connected across a 220 volt source of e.m.f. Which bulb will glow brighter ?***

Solution: $R = \dfrac{V^2}{P}$

$\therefore$ Resistance of first bulb is $R_1 = \dfrac{V^2}{P_1}$,

and resistance of the second bulb is $R_2 = \dfrac{V^2}{P_2}$

In series same current will pass through each bulb

$\therefore$ Power developed across first is $P'_1 = I^2 \dfrac{V^2}{P_2}$

and that across second is $P'_2 = I^2 \dfrac{V^2}{P_2}$

$\Rightarrow \quad \dfrac{P'_1}{P'_2} = \dfrac{P_2}{P_1}$

As $\quad P_2 < P_1 \quad \Rightarrow \quad \dfrac{P_2}{P_1} < 1$

$\Rightarrow \quad \dfrac{P'_1}{P'_2} < 1 \quad \Rightarrow \quad P'_1 < P'_2$

The bulb rated 220 V and 40 W will glow more.

SUMMARY

- Current: An electric current i in a conductor is given by $i = \dfrac{dq}{dt}$
- Current Density: Current is related to current density $\vec{J}$ by $i = \int \vec{J}.d\vec{A}$
- Drift Speed: When an electric field $\vec{E}$ is established in a conductor, the charge carriers aquire a drift speed v_d in the direction of $\vec{E}$. The velocity $\vec{v}_d$ is related to the current density by

  $\vec{J} = (ne)\vec{v}_d$, where ne = carrier charge density.
- **Mobility :** $\mu = m^2v^{-1}s^{-1}$
- Resistance of a conductor: $R = \dfrac{V}{i}$ (definition of R)
- Resistivity and conductivity: $\rho = \dfrac{1}{\sigma} = \dfrac{E}{J}$
- The resistance R of a conducting wire of length L and uniform cross–section is $R = \dfrac{\rho L}{A}$, where A is the cross–sectional area.
- Change of ρ with temperature: $\rho - \rho_0 = \rho_0 \alpha (T - T_0)$

  where T_0 and ρ_0 are reference temperature and resistivity, respectively.

- Ohm's law: A given device obeys Ohm's law if its resistance R = V / i is independent of the applied potential difference V.
- Superconductors: Superconductors lose all electrical resistance at low temperatures.
- **Kirchhoff's law:**
  (a) At any junction of circuit elements, the sum of currents entering the junction must equal the sum of currents leaving it.
  (b) The algebraic sum of changes in potential around any closed loop must be zero.
- **Resistor:**
  (a) Total resistance of n resistors, each of resistance R, connected in series is given by $R = R_1 + R_2 + ... + R_n$
  (b) Total resistance of n resistors connected in parallel is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$
- Wheatstone Bridge: It is the arrangement of four resistances P, Q, R, S. The null point condition is given by $\frac{P}{Q} = \frac{R}{S}$.

FINAL EXERCISE

1. Explain the drift of free electrons in a metallic conductor under external electric field. Derive an expression for drift velocity.
2. Define electric current and discuss Ohm's law.
3. Define resistivity of a conductor. How does the resistance of a wire depend on the resistivity of its material, its length and area of cross-section?
4. Define electrical conductivity. Write its unit. How does electrical conductivity depend on free electron concentration of the conductor?
5. Explain the difference between ohmic and non-ohmic resistances. Give some examples of non-ohmic resistances.
6. The colours on the resistor shown here are red, orange, green and gold as read from left to right. How much is the resistance according to colour code?
7. Three resistors of resistances R_1, R_2 and R_3 are connected (i) in series, and (ii) in parallel. Calculate the equivalent resistance of combination in each case.
8. What is the difference between emf and potential difference between the electrodes of a cell. Derive relation between the two.
9. Explain the difference between primary cells and secondary cells.
10. State Kirchhoff's rules governing the currents and electromotive forces in an electrical network?
11. Give theory of Wheatstone's bridge method for measuring resistances.
12. Discuss the theory of potentiometer.
13. How will you measure unknown potential difference with the help of a potentiometer?
14. Describe potentiometer method of comparing e.m.f. of two cells.
15. How will you determine internal resistance of a cell with the help of a potentiometer? What factors are responsible for internal resistance of a cell ?
16. A wire of length 1 m and radius 0.1 mm has a resistance of 100Ω. Calculate the resistivity of the material.
17. Consider a wire of length 4m and cross-sectional area 1mm^2 carrying a current of 2A. If each cubic meter of the material contains 10^{29} free electrons, calculate the average time taken by an electron to cross the length of the wire.

09 MAGNETISM & MAGNETIC EFFECT OF ELECTRIC CURRENT

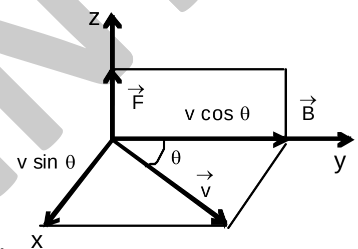
A magnetic field is the space around a magnet or the space around a conductor carrying current in which magnetic influence can be experienced. In the latter case, the magnetic field disappears as soon as the current is switched off. It suggests that motion of electrons in the wire produces a magnetic field. In general, a moving charge is a source of magnetic field.

Consider a positive charge q moving in a uniform magnetic field $\vec{B}$, with velocity $\vec{v}$. Let the angle between $\vec{v}$ and $\vec{B}$ be θ. Due to interaction between the magnetic field produced due to moving charge (i.e. current) and magnetic field applied, the charge q then experiences a force, which depends on the following factors.

(i) $F \propto q$

(ii) $F \propto v \sin\theta$

(iii) $F \propto B$



Combining the above factors, we get

$F \propto qvB \sin\theta$

$F = k\, qvB \sin\theta$

where k is a constant of proportionality. Its value is found to be one, i.e. k = 1

$\therefore F = qvB \sin\theta$

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

$$\vec{F} = q(\vec{v} \times \vec{B})$$

The direction of $\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. It is directed as given by Right Hand Rule.

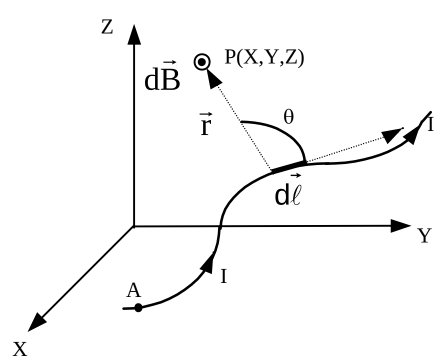
If v = 1, q = 1 and $\sin\theta = 1$,

then $F = 1 \times 1 \times B \times 1 = B$

Thus, the magnetic field induction at any point in the field is equal to the force experienced by a unit charge moving with a unit velocity perpendicular to the direction of magnetic field at that point.

The Biot and Savart Law

Biot–Savart's Law is an experimental law predicted by Biot and Savart. This law deals with the magnetic field induction at a point due to a small current element.



Consider an infinitesimal element of length 'dl' of a wire carrying current I. The magnetic field $d\vec{B}$ at P because of $d\vec{l}$ is given by this law as:

$$d\vec{B} = \frac{\mu_0 I(d\vec{l} \times \vec{r})}{4\pi r^3}$$

Here, $d\vec{l}$ is a vector of length dl which is along the direction of current. The product $Id\vec{l}$ is called current element, it is the smallest possible entity causing a magnetic field.

Note: μ_0 = permeability of free space.

Magnetic field induction at point P due to current through entire wire is

$$\vec{B} = \int \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \vec{r}}{r^3}$$

Biot–Savart's law in terms of current density J, states that

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{J} \times \vec{r}}{r^3} dv$$, where dv is volumetric element.

Biot–Savart's law in terms of charge (q) and its velocity v is

$$d\vec{B} = \frac{\mu_0}{4\pi} q \frac{(\vec{v} \times \vec{r})}{r^3}$$

Biot–Savart's law in terms of magnetising force or magnetic intensity (H) of the magnetic field

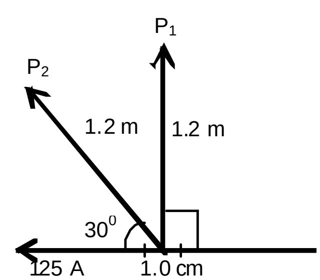
$$d\vec{H} = I \frac{d\vec{l} \times \vec{r}}{r^3}$$

Some important features of BiotJSavart's law:

1. Biot Savart's law is valid for a symmetrical current distribution.
2. Biot Savart's law is applicable only to a very small length of conductor carrying current.
3. This law cannot be easily verified experimentally as the current carrying conductor of very small length cannot be obtained practically.
4. This law is analogus to Coulomb's law in electrostatics.
5. The direction of $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\vec{r}$.
6. If $\theta = 0°$, i.e. point P lies on the axis of the linear conductor carrying current, then $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2} = 0$
7. If $\theta = 90°$, i.e. the point P lies at a perpendicular position w.r.t. to current, then

$$dB = \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$, which is maximum.

Illustration 1. A copper wire carries a steady current of 125 A to an electro-plating tank. Find the magnetic field caused by a 1.0 cm segment of this wire at a point 1.2 m away from it (a) point P_1, straight out to the side of the segment; (b) point P_2, on a line at 30° to the segment.



Solution: (a) $B = \frac{\mu_0}{4\pi} dB = \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \frac{\sin¤}{r^2}$

$$= \frac{10^{-7} \times 125 \times 1 \times 10^{-2} \times \sin 90^0}{(1.2)^2} = 8.7 \times 10^{-8} \text{ T}$$

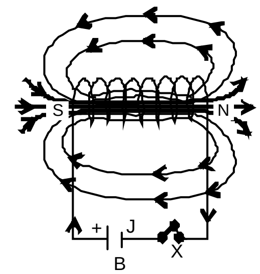
(b) $$B = \frac{10^{-7} \times 125 \times 1 \times 10^{-2} \times \sin 30^0}{(1.2)^2} = 4.35 \times 10^{-8} \text{ T}$$

Direction of magnetic field:

The magnetic field lines due to a straight conductor carrying current are in the form of concentric circles with the conductor as centre, lying in a plane perpendicular to the straight conductor.

The direction of magnetic field lines can be given by right hand thumb rule or Maxwell's cork screw rule.

Magnetic field due to current in a solenoid

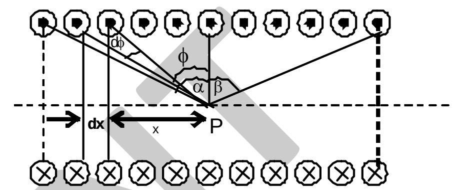


A solenoid consists of an insulated long wire closely wound in the form of a helix. Its length is very large as compared to its distance.

Consider a long straight solenoid having n turns per unit length and carrying current I. The magnetic field setup in the solenoid is as shown.

A linear solenoid carrying current is equivalent to a bar magnet. The magnetic field lines due to current carrying solenoid resemble exactly with those of a bar magnet.

The magnetic field induction at a point just outside the curved face of the solenoid carrying current is zero.



The field at a point on the axis of a solenoid can be obtained by superposition of fields due to a large number of identical coils all having their centre on the axis of solenoid.

Let us consider a coil of width dx at a distance x from the point P on the axis of solenoid as shown in the figure. The magnetic field due to this coil

$$dB = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}}$$

Here, $N = ndx$, $x = R\tan\phi$ and $dx = R\sec^2\phi\, d\phi$

Hence, $$dB = \frac{\mu_0\, ndx \times IR^2}{2(R^2 + R^2\tan^2\phi)^{3/2}}$$

$$B = \int dB = \frac{\mu_0}{2} nI \int_{-\alpha}^{\beta} \cos\phi \qquad \text{i.e.} \qquad B = \frac{\mu_0 nI}{2}[\sin\alpha + \sin\beta]$$

Magnetic field due to current in a toroid

A toroid is an endless solenoid in the form of a ring. According to ampere's circuital law, the line integral of magnetic field induction $\vec{B}$ along the circular path of radius r is given by

$\int \vec{B}.d\vec{l} = \mu_0 \times$ total current passing through circle of radius r.

Now, $\int \vec{B}.d\vec{l} = B(2\pi r)$

Total current passing through circle of radius r = number of turns in the solenoid $\times$ I

$= 2\pi r\, nI$

$\therefore \quad B(2\pi r) = \mu_0 (2\pi rnI)$

or $\quad B = \mu_0 nI$

It is to be noted that the magnetic field B due to a toroid carrying current is independent of r but depends upon the current and number of turns per unit length of toroid. The magnetic field inside the toroid is constant and is always tangential to the circular closed path.

Ampere's Law

Similar to the Gauss's law of electrostatics, this law provides us shortcut methods of finding magnetic field in cases of symmetry. According to this law, the line integral of magnetic field over a closed path $\left(\int \vec{B}.d\vec{l}\right)$ is equal to μ_0 times the net current crossing the area enclosed by that path. Mathematically, $\int \vec{B}.d\vec{l} = \mu_0 i_{enclosed}$

Positive direction of current and the direction of the line integral are given by the right hand thumb and curling fingers, respectively.

In order to find magnetic field using Ampere's law, the closed path of line integral is generally chosen such that $\vec{B}$ is either parallel or perpendicular to the path line. Also, wherever $\vec{B}$ is parallel to the path, its value should be constant.

Magnetic Force on the moving Charge

1. When a charged particle having charge 'q' is projected into a magnetic field, it experiences a magnetic force which is given by the expression

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

Here, $\vec{v}$ = velocity of the particle and $\vec{B}$ = magnetic field

On the basis of above expression, we can draw following conclusion.

(a) Stationary charge (i.e $\vec{v}$ = 0) experiences no magnetic force.

(b) If $\vec{v}$ is parallel or anti parallel to $\vec{B}$, then the charged particle experiences no–magnetic force.

(c) Magnetic force is always perpendicular to both $\vec{v}$ and $\vec{B}$.

(d) As the magnetic force is always perpendicular to $\vec{v}$, it does not deliver power to the charged particle.

(e) As magnetic force is always perpendicular to $\vec{v}$, this force will compel the charged particle to move in a circular path.

2. On the basis of expression $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$, the maximum value of magnetic force is equal to F = qvB, which occurs when the charge is projected perpendicular to the magnetic field. In this case path of charged particle is purely circular (in uniform $\vec{B}$) and magnetic force provides necessary centripetal force.

(a) If radius of the circular path is R then,

$\frac{mv^2}{R} = qvB$, where m = mass of the particle

$$\Rightarrow \qquad R = \frac{mv}{qB}$$

Time taken to complete one revolution is $T = \frac{2\pi R}{v}$

$$\Rightarrow \qquad T = \frac{2\pi m}{qB}$$

3. In general $\vec{v}$ can be resolved into two components, one along the $\vec{B}$ say $v_{\parallel}$ (parallel component) and the other perpendicular to $\vec{B}$ say $v_{\perp}$. Due to $v_{\perp}$ it experiences a magnetic force and hence has a tendency to move on a circular path. Due to $v_{\parallel}$ it experiences no force, and hence has a tendency to move on a straight path along the field. So in this case it moves along a helical path.

(a) Radius of the helix is $R = \frac{mv_{\perp}}{qB}$

(b) Time taken to complete one revolution is $T = \frac{2\pi m}{qB}$. (Note: T is independent of v)

(c) The distance moved by the charged particle along the magnetic field during one revolution is called pitch.

$$\Rightarrow \qquad \text{pitch} = v_{||} \times T = \frac{2\pi m v_{||}}{qB}$$

Lorentz force: The force experienced by a charged particle moving in space where both electric and magnetic fields exists is called Lorentz force.

Force due to electric field, $\vec{F}_e = q\vec{E}$

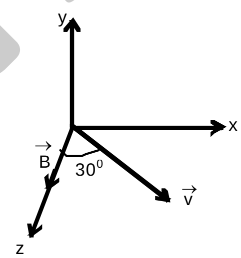
Force due to magnetic field, $\vec{F}_m = q(\vec{v} \times \vec{B})$

Due to both the electric and magnetic fields, the total force experienced by the charged particle will be given by

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{v} \times \vec{B}) = q(\vec{E} + \vec{v} \times \vec{B})$$

This is called Lorentz force.

Illustration 2. A beam of protons moves at 3×10^5 m/s through a uniform magnetic field with magnitude 2.0 T that is directed along the positive z–axis. The velocity of each proton lies in the x–z plane at an angle of 30° to the +z axis. Find the force on a proton.



Solution: Since the charge is positive, so the force is in the same direction as the vector product $\vec{v} \times \vec{B}$

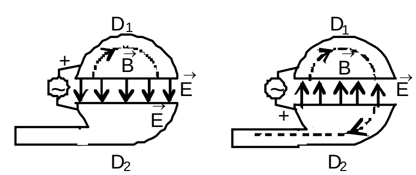
$$\vec{F} = 1.6 \times 10^{-19}(3 \times 10^5 \text{ m/s})\left[\left(\sin 30\hat{i} + \cos 30\hat{k}\right) \times 20\,T\hat{k}\right]$$

$$= -4.8 \times 10^{-14} \text{ N}\hat{j}$$

$$= 4.8 \times 10^{-14} \text{ N}\left(-\hat{j}\right)$$

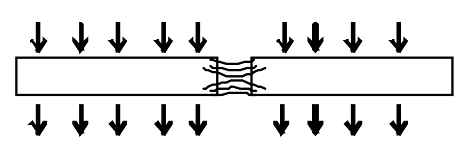
THE CYCLOTRON

In electrostatic accelerators, the acceleration depends on the total potential difference ΔV. To produce high energy particles, ΔV must be very large. However, in a cyclic accelerator an electric charge may receive a series of accelerations by passing many times through a relatively small potential difference.



Essentially, a cyclotron consists of a cylindrical cavity divided into two halves each called dee and placed in a uniform magnetic field parallel to its axis. The two dees are electrically insulated from each other.

An ion source S is placed at the centre of the space between the dees. The system must be maintained within a high vacuum to prevent collisions between the accelerated particles and any gas molecule. An alternating potential difference of the order of 10^4 V is applied between the dees. When the ions are positive, they will be accelerated towards the negative dee. once the ions get inside a dee, they experience no electrical force, since the electric field is zero in the interior of a conductor. However, the magnetic field makes the ions describe a circular orbit with a radius given by $r = \frac{mv}{qB}$



and angular velocity $\omega = qB/m$

In this way, the potential different between dees is in resonance with the circular motion of the ions.

As the ion describes half a revolution, the polarity of dees is reversed. When the ions cross the gap between them. They receive another small acceleration. The next half-circle described then has a large radius but the same angular velocity. The process repeats itself several times, until the radius attains maximum value R, which is practically equal to the radius of the dees. The poles of the magnet are designed. Such that the magnetic field at the edge of dees decreases sharply and the ions move tangentially, escaping through a convenient opening. The maximum velocity v_{max} is related to radius R by

$$R = \frac{mv_{max}}{qB}$$

or $\quad v_{max} = (q/m)BR$

$$\Rightarrow \quad E_k = \frac{1}{2}mv_{max}^2 = \frac{1}{2}q\left(\frac{q}{m}\right)B^2R^2 .$$

Illustration 3. A magnetron in a microwave oven emits electromagnetic waves with frequency f = 2450 MHz. What magnetic field strength is required for electrons to move in circular paths with this frequency?

Solution: $\quad B = \dfrac{m\omega}{q} = \dfrac{2\pi fm}{q}$

$$= \frac{2\times\pi\times 2450\times10^{6}\times9.11\times10^{-31}}{1.6\times10^{-19}} = 0.0877\,\text{T}$$

Force on a Current Carrying Wire

As $\quad \vec{F} = q(\vec{v}\times\vec{B})$,

We can say, $\quad d\vec{F} = dq(\vec{v}\times\vec{B})$

or $\quad d\vec{F} = dq\left[\dfrac{d\vec{l}}{dt}\times\vec{B}\right] = \dfrac{dq}{dt}\left[d\vec{l}\times\vec{B}\right]$

$\Rightarrow \quad d\vec{F} = I(d\vec{l}\times\vec{B})$

FLEMING'S LEFT-HAND RULE

In the special case of straight wire of length ℓ in a uniform magnetic field B, $\vec{F} = I(\vec{l}\times\vec{B})$

The direction of the force $\vec{F} = I(\vec{l}\times\vec{B})$ is given by the Fleming's left hand rule.

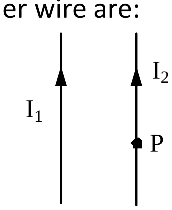
Close your left fist and then shoot your index finger in the direction of the magnetic field. Relax your middle finger in the direction of the current. The force on the conductor is shown by the direction of the erect thumb.

Force between two infinite parallel current carrying wires.

Let two infinite parallel wires carrying currents I_1 and I_2 be separated by a distance r.

If we take an arbitrary point. 'P' on the second wire, the angle α and β subtended by the other wire are:

$\alpha = 0$ and $\beta = 0$.



$$B_{21} = \frac{\mu_0 I_1}{4\pi r}[\cos 0 + \cos 0] = \frac{\mu_0 I_1}{2\pi r}$$

$$\vec{F}_{21} = I_2\left(\vec{l}\times\vec{B}_1\right)$$

$$\Rightarrow \qquad \frac{I_2 l_2 \mu_0 I_1}{2\pi r}$$

$$\frac{F_{21}}{l_2} = \frac{\mu_0 I_1 I_2}{2\pi r} \qquad \text{and by symmetry } \frac{F_{12}}{l_1} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Force per unit length = $\frac{\mu_0 I_1 I_2}{2\pi r}$

Note that wires carrying current in the same direction attract each other.

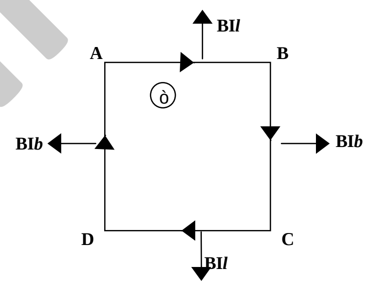
If $I_1 = I_2 = 1$ amp; r = 1 m

Then, $F = \frac{\mu_0}{4\pi} \times \frac{2 I_1 I_2}{r} = 10^{-7} \times 2 \times 1 \times 1 = 2 \times 10^{-7}$ N/m

Thus, one ampere is that much current which when flowing through each of the two parallel uniform long linear conductors placed in free space at a distance of one meter from each other will attract or repel each other with a force of 2×10^{-7} N/m of their lengths.

Torque on a current carrying planer loop in a uniform magnetic field

Case I:



When plane of the loop is perpendicular to magnetic field

Length of AB = DC = ℓ and that of BC = AD = b

Forces experienced by all the sides are shown in the figure.

$\therefore$ Forces on AB and DC are equal and opposite to the each other same is the case with BC and AD.

$\Rightarrow \qquad \Sigma F = 0$

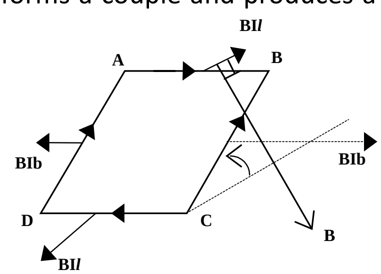
Since the line of action of the forces on AB and DC is same and also the line of action of the forces BC and AD is same, therefore, torque is zero.

Case II:

When the plane of the loop is inclined to the magnetic field.

In this case again $\Sigma F = 0$

$\therefore$ Lines of action of the forces on AB and DC are different, therefore, this forms a couple and produces a torque. Side view of the loop is shown in the figure.



Torque = $BIl\,(b\sin\theta) = BI(lb)\sin\theta = BIA\sin\theta$.

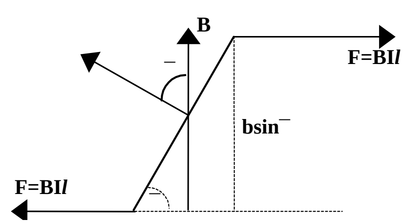
If loop has N turns, then $\tau = BNIA \sin\theta$

In vector form, $\vec{\tau} = \vec{\mu} \times \vec{B}$, where $\vec{\mu} = NI\vec{A}$

Energy needed to rotate the loop through an angle dθ is

$dU = \tau d\theta$

$$\Rightarrow \Delta U = \int dU = \int_{\theta_2}^{\theta_2} \tau\, d\theta = \int_{\theta_1}^{\theta_2} \mu B \sin\theta\, d\theta$$



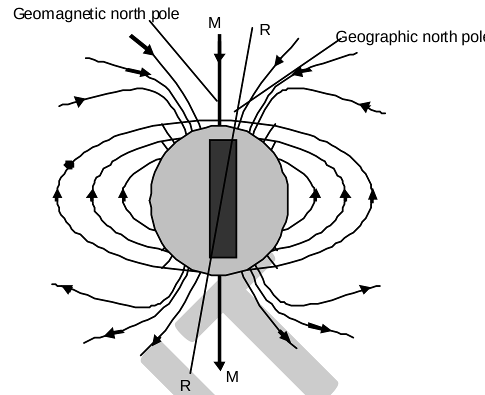
$\Delta U = \mu B(\cos\theta_1 - \cos\theta_2)$,

if we choose θ_1 such that at $\theta = \theta_1$, $U_1 = 0$

This is the energy stored in the loop, $U = -\vec{\mu}_m . \vec{B}$

THE MAGNETISM OF EARTH

Earth is a huge magnet; for points near earth's surface, its magnetic field can be approximated as the field of a huge bar magnet –a magnetic dipole that straddles the centre of the planet. The figure shown is an idealized symmetric depiction of the dipole field, without the distortion caused by passing charged particles from the sun.



Because earth's magnetic field is that of a magnetic dipole, a magnetic dipole moment $\vec{\mu}$ is associated with the field. For the idealized field, the magnitude of $\vec{\mu}$ is 8×10^{22} J/T and the direction of $\vec{\mu}$ makes an angle of 11.5° with the rotation axis (RR) of the earth. The dipole axis MM lies along $\vec{\mu}$ and intersects earth's surface at the geometric north pole in north west Greenland and the geometric south pole in Antarctica. The lines of the magnetic field $\vec{B}$ generally emerges in the southern hemisphere, and re–enter earth in the northen hemisphere.

The direction of the magnetic field at any location on earth's surface is commonly specified in terms of two angles. The field declination is the angle (left or right) between geographic north (which is towards 90° latitude) and the horizontal component of the field. The field inclination is the angle (up or down) between a horizontal plane and the field's direction.

Magnetism and electrons

Magnetic materials are magnetic because of the electron within them. One way in which electrons can generate magnetic field is to send them through a wire as an electric current and their motion produces a magnetic field around the wire.

Spin magnetic dipole moment

An electron has an intrinsic angular momentum called its spin angular momentum or just spin $\vec{s}$; associated with the spin is an intrinsic spin magnetic dipole moment $\vec{\mu}_s$. $\vec{s}$ and $\vec{\mu}_s$ are related by

$$\vec{\mu}_s = -\frac{e}{m}\vec{s}$$

where e and m are charge and mass of electron, respectively. The minus sign means that $\vec{\mu}_s$ and $\vec{s}$ are oppositely directed.

Spin $\vec{s}$ is different from the angular momentum:

- $\vec{s}$ itself cannot be measured. However, its component along any axis can be measured.
- A measured component of $\vec{s}$ is quantized, which is a general term that means it is restricted to certain values. A measured component of $\vec{s}$ can have only two values which differ only in sign.

Let us assume that component of spin $\vec{s}$ is measured along the z–axis of a coordinate system. Then, the measured component $S_z = m_s \frac{h}{2\pi}$ for $m_s = \pm\frac{1}{2}$

where m_s is called the spin magnetic quantum number.

When S_z is parallel to the z–axis, m_s is $+\frac{1}{2}$ and the electron is said to be spin up. When S_z is antiparallel to the z–axis, m_z is $-\frac{1}{2}$ and the electron is said to be spin down.

We can relate the component

$$\mu_{s,z} = -\frac{e}{m}s_2 = \pm\frac{eh}{4\pi m}$$

where the plus and minus signs corresponds to $\mu_{s,z}$ being parallel and antiparallel to the z–axis, respectively. This quantity is termed as Bohr magneton

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \text{J/T}$$

Orbital magnetic dipole moment

When it is an atom, an electron has an additional angular momentum called its orbital angular momentum $\vec{L}_{orb.}$. Associated with $\vec{L}_{orbt}$ is an orbital magnetic dipole moment $\vec{\mu}_{orbt}$,

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}$$

The minus sign means that $\vec{\mu}_{orbt}$ and $\vec{L}_{orbt}$ have opposite directions.

Here, again $L_{orb,z} = m_L\frac{h}{2\pi}$ [for m_L = 0, ± 1, ±2 . . . . ± (limit)]

in which m_L is called the orbital magnetic quantum number and limit refers to some larger allowed integer value of m_L.

We can write the z– component $\mu_{orb,z}$ of the orbital magnetic dipole moment as $\mu_{orb,z} = -m_L\frac{e}{4\pi m}$

and in terms of Bohr magneton as $\mu_{orb,z} = -m_L\mu_B$

Magnetic Materials

Each electron in an atom has an orbital magnetic dipole moment and a spin magnetic dipole moment that combine vectorially. The resultant of these two vector quantities combines vectorially with similar resultants for all other electrons in the atom, and the resultant for each atom combines with those for all these magnetic dipole moments produces a magnetic field, the material is magnetic. There, are three general types of magnetism : diamagnetism; paramagnetism and ferromagnetism.

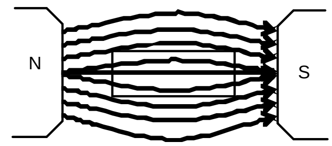
1. Paramagnetism is exhibited by materials containing transition elements, rare earth elements and actinide elements. Each atom of such a material has a permanent resultant magnetic dipole moment, but the moments are randomly oriented in the material and material as a whole lacks a net magnetic field. However, an external magnetic field $\vec{B}_{ext}$ can partially align the atomic magnetic dipole moments to give the material a net magnetic field. The alignment and thus its field disappear when $\vec{B}_{ext}$ is removed. The term paramagnetic material usually refers to materials that exhibit primarily paramagnetism.

2. In diamagnetism, weak magnetic dipole moments are produced in the atoms of the material when the material is placed in an external magnetic field $\vec{B}_{ext}$; the combination of all those induced dipole moments and thus their net field disappear when $\vec{B}_{ext}$ is removed. The term diamagnetic material usually refers to materials that exhibit only diamagnetism.

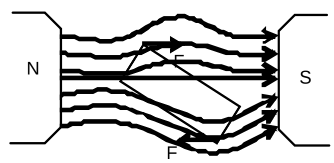
3. Ferromagnetism is a property of iron, nickel and certain other elements (and of compounds and alloys of these elements). Some of the electrons in these materials have their resultant magnetic dipole moment aligned, which produces regions with strong magnetic dipole moments. An external field $\vec{B}_{ext}$ can then align the magnetic moments of such regions, producing a strong magnetic field for a sample of the material; the field partially persists when $\vec{B}_{ext}$ is removed. The term ferromagnetic material and even the common term magnetic material is used to refer to materials that exhibit primarily ferromagnetism.

Paramagnetic substances:

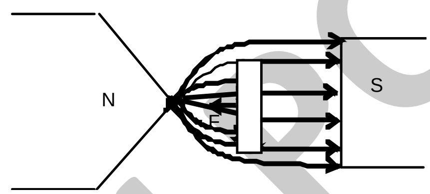
(i) The substance, when placed in magnetic field, acquires a very feeble magnetisation in the same sense as the applied field. Thus the magnetic induction inside the substance is slightly greater than outside.



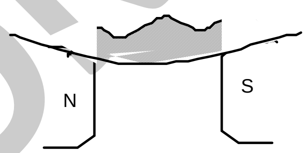
(ii) In a uniform magnetic field, these substances rotate until longest axis are parallel to the field.



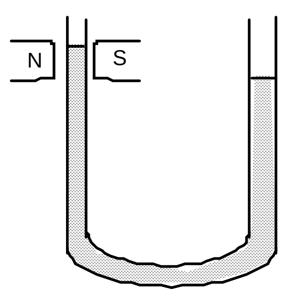
(iii) In non–uniform magnetic field, these substances are attracted towards stronger magnetic field.



If a paramagnetic liquid is placed in a watch glass resting on the poles of powerful electromagnets, the liquid is found to move so that the general depth at a points of greatest magnetic field.



(iv) If a paramagnetic material (liquid) is filled in a narrow U–tube and one limb is placed in between the pole pieces of an electromagnet such that the level of the liquid is in line with the field, then the liquid will rise as the field is switched on.

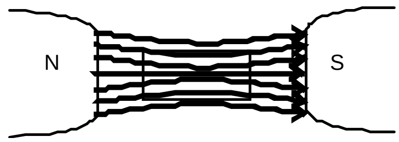


(v) The relative permeability k is slightly greater than one.

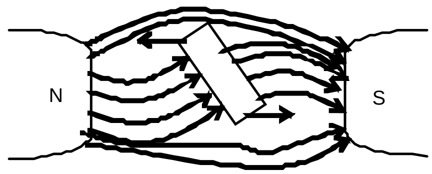
(vi) The susceptibility χ does not change with magnetising field at a particular temperature. But as temperature increases, χ decreases, i.e. it varies inversely as the absolute temperature.

Diamagnetic Substances:

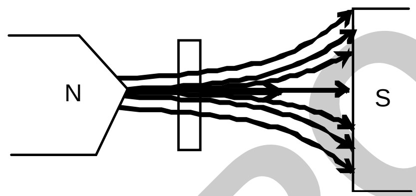
(i) These substances, when placed in a magnetic field, acquire feeble magnetisation in a direction opposite to applied field. Thus, the lines of induction inside the substance are smaller than outside it.



(ii) In a uniform magnetic field, these substances rotate until their longest axes are normal to the field.



(iii) . . . . . . –uniform field, these substances move from stronger to weaker parts of the field.



If a diamagnetic liquid is placed in a watch glass resting on the pole of a powerful electromagnet, the liquid is found to accumulate on sides, where the field is weaker.

(iv) If a diamagnetic liquid is filled in a narrow U–tube and one–limb is placed between the pole pieces of a electromagnet, the level of the liquid depresses as and when the magnetic field is switched on.

(v) The relative permeability k is slightly less than 1.

(vi) The susceptibility χ of such substances is always negative. It is constant and does not vary with field or the temperature.

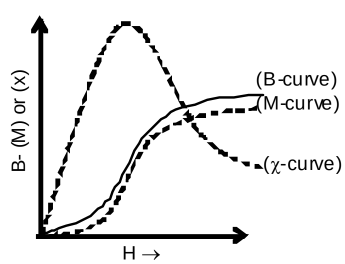
Ferromagnetic substances:

(i) These substances are strongly magnetised by even a weak magnetic field.

(ii) The relative permeability is very large and is of the order of thousands even.

(iii) The susceptibility is positive and very large.

(iv) The intensity of magnetization M is proportional to the magnetising field intensity H for its smaller values, increases rapidly for larger values and attains a constant value for large values of H.



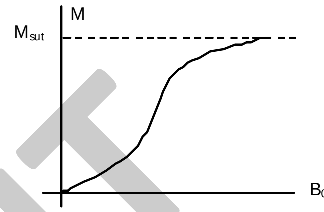
(v) Permeability μ also varies as χ except at very high magnetic fields where μ decreases slowly in comparison to χ.

Hysteresis curves:

In the ferromagnetic materials, strong interactions between atomic magnetic moments cause them to line up parallel to each other in regions called magnetic domains, even when no external field is present.

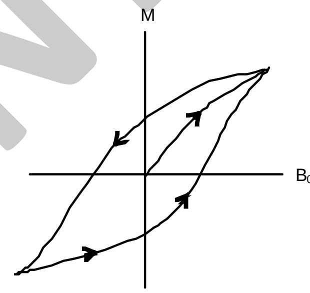
When there, is no externally applied field, the domain magnetization are randomly oriented. Application of $\vec{B}_0$, makes the domain to orient themselves parallel to the field.

As the external field is increased, a point is eventually reached out at which nearly all the magnetic moments in the materials aligns parallel to the external field. This condition is called the saturation magnetization.



For many ferromagnetic materials, the relation of magnetization to external magnetic field is different when the external field is increasing from when it is decreasing.

In the given example, when the material is magnetized to saturation and then the external field is reduced to zero some magnetization remains. This behaviour is characteristic of permanent magnets, which retain most of their saturation magnetization when the magnetic field is removed. To reduce magnetization to zero requires a magnetic field in reverse direction.



This behaviour is called hysteresis and the curves are called hysteresis loops. Magnetization and demagnetisation of a material involves dissipation of energy by heat loss.

Soft and hard magnetic materials:

The magnetic properties of a ferromagnetic substance can be obtained from the size and shape of the hysteresis loop. The study of curve for different materials gives the following information.

(i) **Susceptibility:** The susceptibility is the intensity of magnetisation per unit magnetisizing field (i.e. M/H) is greater for soft materials than for hard materials.

(ii) **Permeability:** The permeability, the magnetic induction per unit magnetizing field, (i.e. B/H) is greater for soft materials.

(iii) **Retentivity:** When a magnetic specimen is first magnetised and then the magnetising field is reduced to zero, the specimen retains intensity of the magnetisation (or the magnetic induction). This is known as retentivity. It is greater for soft material.

(iv) **Coercivity:** To demagnetise the magnetic specimen completely a –ve field is required. The value of reverse field H required to reduce the intensity of magnetisation to zero is known as the correcivity. It is less for soft materials.

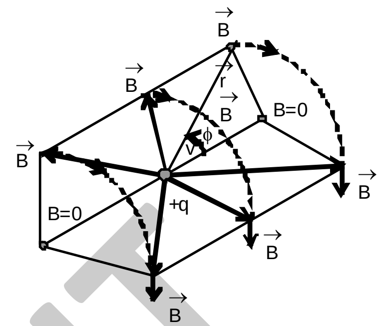
Choice of magnetic materials:

The choice of a magnetic material for different uses can be decided from hysteresis curves of a specimen of the material.

(i)	**Permanent magnets**	**Electromagnets**	**Choke coil, Armature coil and motors**
Retentivity	High	–	–
Coactivity	High	–	–
Hysteresis loss	Immaterial	Low	Low
Permeability	–	High	High
Sp. resistance	–	–	High
Example	Steel, vicalloy	Iron	copper

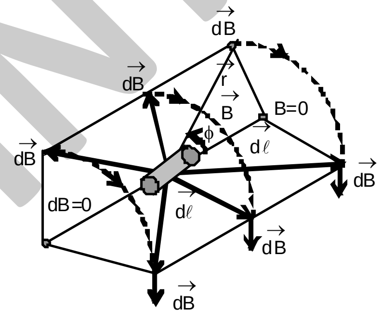
SUMMARY

- The magnetic field $\vec{B}$ created by a charge q moving with velocity $\vec{v}$ depends on the distance r from the source point (the location of q) to the field point (where $\vec{B}$ is measured). the field $\vec{B}$ is perpendicular to $\vec{v}$ and to $\hat{r}$, the unit vector directed from the source point to the field point. The principle of superposition of magnetic fields states that the total field produced by several moving charges is the vector sum of the fields produced by the individual charges.



$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^2}$$

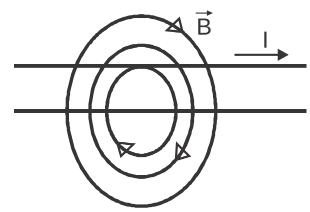
- The law of Biot and Savart gives the magnetic field $d\vec{B}$ created by an element $d\vec{\ell}$ of a conductor carrying current I.



$$\vec{B} = \frac{\mu_0}{4\pi}\frac{Iq\vec{l}\times\vec{r}}{r^2}$$

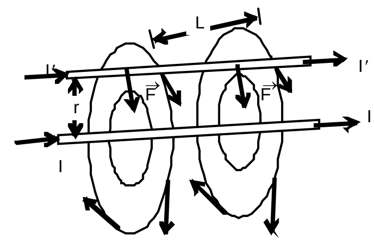
- The field $d\vec{B}$ is perpendicular to both $d\vec{\ell}$ and $\hat{r}$, the unit vector from the element to the field point. The field $\vec{B}$ created by a finite current carrying conductor is the integral of $d\vec{B}$ over the length of the conductor.

- The magnetic field $\vec{B}$ at a distance r from a long, straight conductor carrying a current I has a magnitude that is inversely proportional to r. The magnetic field lines are circles coaxial with the wire, with directions given by the right hand rule.



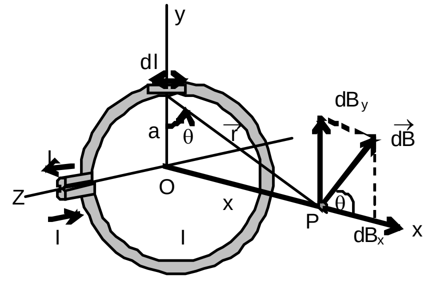
$$B = \frac{\mu_0 I}{2\pi r}$$

- Two long, parallel, current–carrying conductors attract if the currents are in the same direction and repel if the currents are in opposite directions. The magnetic force per unit length between the conductors depends on their currents I and I′ and their separation r. The definition of the ampere is based on this relation.



$$\frac{F}{L} = \frac{\mu_0 II'}{2\pi r}$$

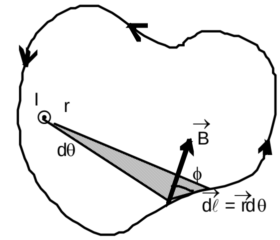
- The law of Biot and Savart allows us to calculate the magnetic field produced along the axis of a circular conducting loop of radius a carrying current I. The field depends on the distance x along the axis from the centre of the loop to the field point. If there, are N loops, the field is multiplied by N. At the center of the loop, x = 0.



$$B_x = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}} \qquad B_x = \frac{\mu_0 N I}{2a}$$

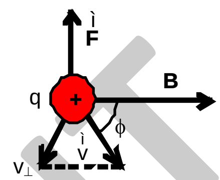
(Circular loop) (Centre of N circular loops)

- Ampere's law states that the line integral of $\vec{B}$ around any closed path equals μ_0 times the net current through the area enclosed by the path. The positive sense of current is determined by a right–hand rule.



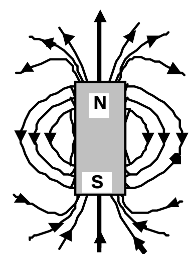
$$\int \vec{B}.d\vec{l} = \mu_0 I_{encl}$$

- Magnetic interactions are fundamentally interactions between moving charged particles. These interactions are described by the vector magnetic field, denoted by $\vec{B}$. A particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{v}$ and $\vec{B}$. The SI unit of magnetic field is tesla (1T = 1 N/A. m)

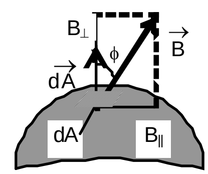


$$\vec{F} = q\vec{v} \times \vec{B}$$

- A magnetic field can be represented graphically by magnetic field lines. At each point a magnetic field line is tangent to the direction of $\vec{B}$ at that point. Where field lines are close together the field magnitude is large, and viceversa.



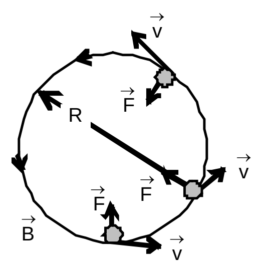
- Magnetic flux ϕ_B through an area is defined in an analogous way to electric flux. The SI unit of magnetic flux is weber. The net magnetic flux through any closed surface is zero. As a result, magnetic field lines always close on themselves.



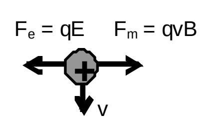
$$\phi_B = \int B_\perp dA = \int B\cos\phi \; dA = \int \vec{B}.d\vec{A}$$

$$\int \vec{B}.d\vec{A} = 0 \text{(closedsurface)}$$

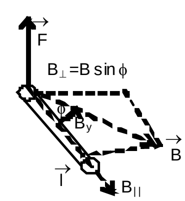
- The magnetic force is always perpendicular to $\vec{v}$; a particle moving under the action of a magnetic field alone moves with constant speed. In a uniform field, a particle with initial velocity perpendicular to the field moves in a circle with radius R that depends on the magnetic field strength B, the particle mass m, speed v, and charge.

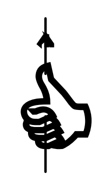


$$R = \frac{mv}{|q|B}$$



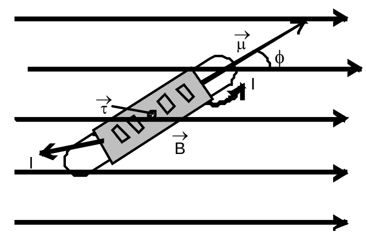
- Crossed electric and magnetic fields can be used as velocity selector. The electric and magnetic forces exactly cancel when v = E/B.
- A straight segment of a conductor carrying current I in a magnetic field B experiences a force F that is perpendicular to both B and the unit vector $\vec{\ell}$, which points in the direction of the current and has magnitude equal to the length of the segment. A similar relationship gives the force $d\vec{F}$ on an infinitesimal current–carrying segment $d\vec{\ell}$.





$$\vec{F} = I\vec{l} \times \vec{B} \qquad d\vec{F} = Id\vec{l} \times \vec{B}$$

- A current loop with area A and current I in a uniform magnetic field $\vec{B}$ experiences no net magnetic force, but does experience a magnetic torque of magnitude τ. The vector torque $\vec{\tau}$ can be expressed in terms of the magnetic moment $\vec{\mu} = I\vec{A}$ of the loop, as can be potential energy U of a magnetic moment in a magnetic field $\vec{B}$. The magnetic moment of a loop depends only on the current and the area; it is independent of the shape of the loop.



  $\tau = IBA \sin\phi$

  $\vec{\tau} = \vec{\mu} \times \vec{B}$

  $U = -\vec{\mu}.\vec{B} = -\mu B \cos$¤

- The following table lists magnetic fields caused by several current distributions. In each case the conductor is carrying current I.

Current Distribution	**Point in Magnetic field**	**Magnetic Field Magnitude**
Long, straight conductor	Distance r from conductor	$B = \frac{\mu_0 I}{2\pi r}$
Circular loop of radius a	On axis of loop At centre of loop	$B = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$ $B = \frac{\mu_0 I}{2a}$ (for N loops, multiply these expression by N)
Long cylindrical conductor of radius R	Inside conductor, r < R Outside conductor r > R	$B = \frac{\mu_0 I r}{2\pi R^2}$ $B = \frac{\mu_0 I}{2\pi r}$
Long, closely wound solenoid with n turns per unit length, near its midpoint	Inside solenoid, near centre Outside solenoid	$B = \mu_0 n I$ $B \approx 0$
Tightly wound toroidal solenoid (toroid) with N turns	Within the space enclosed by the windings, distance r from symmetry axis. Outside the space enclosed by the windings.	$B = \frac{\mu_0 N I}{2\pi r}$

When magnetic material are present, the magnetization of the material causes an additional contribution to $\vec{B}$. For paramagnetic and diamagnetic materials, μ_0 is replaced in magnetic – field expressions by $\mu = K_m\mu_0$, where μ is the permeability of the material and K_m is its relative permeability. The magnetic susceptibility X_m is defined as $X_m = K_m - 1$. Magnetic susceptibilities for paramagnetic materials are small positive quantities; those for diamagnetic material are small negative quantities. For ferromagnetic materials, K_m is much larger than unity and is not constant. Some ferromagnetic materials are permanent magnets, retaining their magnetization even after the external magnetic field is removed.

FINAL EXERCISE

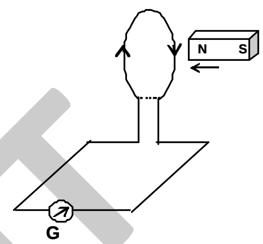
1. The magnetic force between two poles is 80 units. The separation between the poles is doubled. What is the force between them?
2. The length of a bar magnet is 10 cm and the area of cross-section is 1.0 cm^2. The magnetization I = 10^2 A/m. Calculate the pole strength.
3. Two identical bar magnets are placed on the same line end to end with north pole facing north pole. Draw the lines of force, if no other field is present.
4. The points, where the magnetic field of a magnet is equal and opposite to the horizontal component of magnetic field of the earth, are called neutral points

   (a) Locate the neutral points when the bar magnet is placed in magnetic meridian with north pole pointing north.

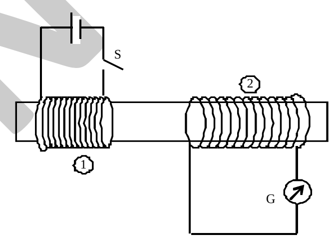
   (b) Locate the neutral points when a bar magnet is placed in magnetic meridian with north pole pointing south.
5. If a bar magnet of length 10 cm is cut into two equal pieces each of length 5 cm then what is the pole strength of the new bar magnet compare to that of the old one.
6. A 10 cm long bar magnet has a pole strength 10 A.m. Calculate the magnetic field at a point on the axis at a distance of 30 cm from the centre of the bar magnet.
7. How will you show that a current carrying conductor has a magnetic field around it? How will you find its magnitude and direction at a particular place ?
8. A force acts upon a charged particle moving in a magnetic field, but this force does not change the speed of the particle, Why ?
9. At any instant a charged particle is moving parallel to a long, straight current carrying wire. Does it experience any force ?
10. A current of 10 ampere is flowing through a wire. It is kept perpendicular to Magnetism a magnetic field of 5T. Calculate the force on its 1/10 m length.
11. A long straight wire carries a current of 12 amperes. Calculate the intensity of the magnetic field at a distance of 48 cm from it.
12. Two parallel wire, each 3m long, are situated at a distance of 0.05 m from each other. A current of 5A flows in each of the wires in the same direction. Calculate the force acting on the wires. Comment on its nature ?
13. The magnetic field at the centre of a 50cm long solenoid is 4.0×10^{-2} $NA^{-1}m^{-1}$ when a current of 8.0A flows through it, calculate the number of turns in the solenoid.
14. Of the two identical galvanometer one is to be converted into an ammeter and the other into a milliammeter. Which of the shunts will be of a larger resistance ?
15. The resistance of a galvanometer is 20 ohms and gives a full scale deflection for 0.005A. Calculate the value of shunt required to change it into an ammeter to measure 1A. What is the resistance of the ammeter ?
16. An electron is moving in a circular orbit of radius 5×10^{-11} m at the rate of 7.0×10^{15} revolutions per second. Calculate the magnetic field B at the centre of the orbit.
17. Calculate the magnetic field at the centre of a flat circular coil containing 200 turns, of radius 0.16m and carrying a current of 4.8 ampere.

10 ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

Electromagnetic Induction



If we bring a magnet near a coil connected with a galvanometer, the galvanometer shows deflection indicating flow of current in the coil. This current flow as long as the magnet is moving, i.e. the magnetic flux through the coil is changing. Once the magnet becomes stationary, the current stops.



If we take two coils wound on an iron cylinder, one fitted with a battery and a switch, the other fitted with a galvanometer, when switch is closed, current increases from zero to its maximum value. During this brief time, the galvanometer deflects showing flow of current. The changing currents in coil (1) gives rise to a changing magnetic field which induces an emf and a current in the coil (2). This phenomenon of inducing electricity by changing magnetic field is known as electromagnetic induction.

FARADAY'S LAWS

(i) When the flux of magnetic induction through a loop is changing, an electromotive force (emf) is induced in the loop. It lasts as long as the magnetic flux changes.

(ii) This induced e.m.f. is equal to the negative rate of change of flux, i.e.,

$$\varepsilon = \frac{-d\Phi}{dt}. \quad \text{where } \Phi = n\int \vec{B}.d\vec{S}, \text{ n = number of turns}$$

$\vec{B}$ = magnetic induction , $d\vec{S}$ = area element

If the resistance of the loop is R, the current in the loop will be $i = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\Phi}{dt}$

Illustration 1. A conducting circular loop having a radius of 5.0 cm, is placed perpendicular to a magnetic field of 0.50 T. It is removed from the field in 0.50 s. Find the average emf produced in the loop during this time.

Solution: Radius = 5 cm

$\therefore$ Area (S) = $25\,\pi \times 10^{-4}$ m^2

$\phi = \vec{B}.\vec{S}$ = BS cos 0^0 = 3.927×10^{-3}

$\therefore$ Average induced emf = $\frac{\Delta\Phi}{\Delta t} = 7.85 \times 10^{-3}$ volt

Illustration 2. A coil of area 500 cm^2 having 1000 turns is placed such that the plane of the coil is perpendicular to a magnetic field of magnitude 4×10^{-5} weber/m^2. If it is rotated by 180° about an axis passing through one of its diameter in 0.1 sec, find the average induced emf.

Solution: Total flux through the loop is

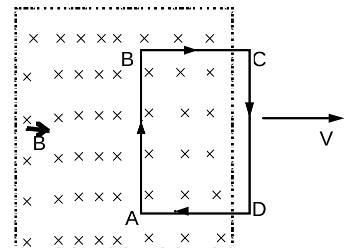
ϕ = B.NA = $4 \times 10^{-5} \times 1000 \times 500 \times 10^{-4}$

Since loop is rotated by 180°

Total change in flux = 2ϕ

i.e. emf = 2ϕ / 0.1 = change in flux / time = $\frac{2 \times 4 \times 5 \times 10^{-4}}{0.1} = 40$ mv

Lenz's law



It states that the polarity of the induced e.m.f. and the direction of induced current is such that it opposes the very cause which produces it. Consider the figure shown. A rectangular loop ABCD is being pulled out of the magnetic field directed into the plane of the paper and perpendicular to the plane of the paper. As the loop is dragged out of the field the flux associated with the loop which is directed into the plane of the paper decreases.

The induced current will flow in the loop in the sense to oppose the decreasing of this flux. For this to happen magnetic field due to induced current in the loop must be directed into the plane of the paper. Hence the current in the loop must flow in the clockwise sense.

Illustration 3. A square–shaped copper coil has edges of length 50 cm and contains 50 turns . It is placed perpendicular to a 1.0 T magnetic field. It is removed from the magnetic field in 0.25 s and restored in its original place in the next 0.25 s. Find the magnitude of the average emf induced in the loop during

(a) Its removal,

(b) its restoration

(c) its entire motion in the field

Solution: No. of turns = 50, Area of square = 0.25 m^2

Magnitude of magnetic field = 1 T

(a) Average rate of complete removal $= \dfrac{\Delta S}{\Delta t} = 1\,m/s^2$

$\therefore$ average induced emf $= NB\dfrac{\Delta S}{\Delta t} = 50\,\text{volt}$

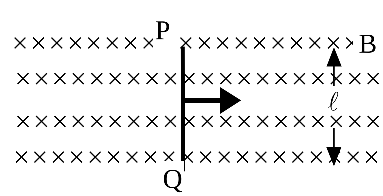
(b) Average rate of complete restoration = $\dfrac{\Delta S}{\Delta t} = 1\,m/s^2$

$\therefore$ average induced emf $NB\dfrac{\Delta S}{\Delta t} = 50\,\text{volt}$

(c) For entire motion in the field $\Delta S = 0$

$\therefore$ average induced emf $= NB\dfrac{\Delta S}{\Delta t} = 0\,\text{volt}$

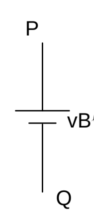
Motional e.m.f.



Consider a straight conductor PQ moving in a magnetic field. The electrons inside it experience a force $\vec{F} = -e\vec{v}\times\vec{B}$ and accumulate at the end of the conductor near Q. Thus, an electric field is established across its ends. Then $-e\vec{v}\times\vec{B}$ is balanced by $-e\vec{E}$ in the opposite direction, at equilibrium.

$\Rightarrow$ $$-e\vec{v}\times\vec{B} - e\vec{E} = 0$$

$$\varepsilon = -\int \vec{E}.d\vec{l} = \int (\vec{v}\times\vec{B}).d\vec{l} = \int \vec{B}.(d\vec{l}\times\vec{v})$$



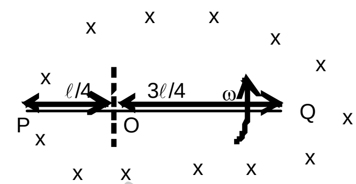
As $d\vec{l}\times\vec{v}$ is the area swept per unit time by length dℓ and hence $\vec{B}.(d\vec{\ell}\times\vec{v})$ is the flux of induction through this area. Therefore, the motional emf is equal to the flux of induction cut by the conductor per unit time. If the ℓ, B and v are mutually perpendicular to each other then £ = Bℓv

We can replace the moving rod by a battery of emf $vB\ell$ with positive terminal at P and the negative terminal at Q.

Illustration 4. A conducting rod of length ℓ is rotating with constant angular velocity ω about point O in a uniform magnetic field B as shown in the figure. The emf induced between ends P and Q will be

(A) $\frac{1}{4}B\omega\ell^2$ (B) $\frac{5}{10}B\omega\ell^2$

(C) zero (D)



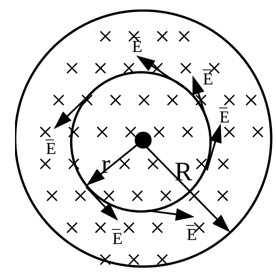
Solution: **(A)** $\varepsilon = \int_{-\ell/4}^{3\ell/4} B\omega x dx = \frac{1}{4}B\omega\ell^2$

Fleming's right hand rule for direction of induced emf

Stretch your right hand thumb, the index finger and middle finger such that all the three are mutually perpendicular to each other. If the thumb represents the direction of the motion of conductor, the index finger the direction of magnetic field, then the middle finger represents the direction of the current.

Time varying magnetic field

If a conducting loop is placed in a time varying magnetic field, this changing magnetic field acts as a source of electric field and hence induces an emf; infact the electric field is induced even when no conductor is present. Faraday discovered that

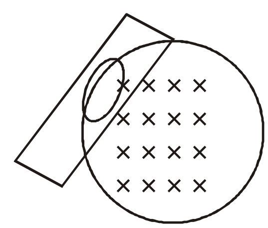


$$\int \vec{E}.d\vec{l} = \frac{d\phi}{dt}$$

This field $\vec{E}$ differs from an electrostatic field, it is non–conservative. We can not define the potential corresponding to this field in the usual sense i.e. $dV = \vec{E}.d\vec{r}$ does not hold here.

$\vec{E}$ has to have a direction shown when $\vec{B}$ is increasing, because $\int \vec{E}.d\vec{l}$ has to be negative when $\frac{d\phi}{dt}$ is positive.

Eddy currents: If a metal plate, e.g. copper is passed through a magnetic field (see figure). During entry into the field and exit from the field, the magnetic flux through a loop (Consider an arbitrary loop as shown in the figure) changes. This change in flux cause current to be set up in the loop. There may be many such loops and currents will flow through them. These are called eddy currents.



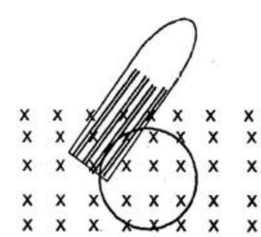
Eddy currents flow in many loops in a plate and cause heating. This thermal energy is produced by conversion of kinetic energy and thus the plate slows down. This is called electromagnetic damping.

To avoid eddy currents, slots are cut in the plate due to which the flow of eddy current is broken

To reduce the losses due to eddy currents, conducting parts are made in large number of thin layers , separated by lacqueran insulator. These are called laminations. These break paths of eddy currents.

Uses of eddy currents:

1. Used for braking systems in trains.
2. For electromagnetic shielding.
3. Used in speedometers
4. Used in induction furnaces



Self Induction

A changing current in a circuit causes a change in the magnetic flux associated with itself, which induces an opposing e.m.f. in it. The net magnetic flux linked with itself is proportional to the current in the loop.

Thus, $\phi = Li$

Where L is a constant called coefficient of self–induction or self inductance. Also e.m.f induced in loop is

$$\varepsilon = -L\frac{di}{dt}$$

The induced emf in case of self inductance opposes the change in the current. Physically it is analogous to inertia in mechanics.

Illustration 5. An average induced emf of 0.20 V appears in a coil when the current in it is changed from 5.0A in one direction to 5.0A in the opposite direction in 0.20s. Find the self-inductance of the coil.

Solution: As $\varepsilon = -L\frac{di}{dt}$

$$\frac{di}{dt} = \frac{(-5.0)-(5.0)}{0.20} = -50\ \text{A/s}$$

Self inductance $L = -\frac{\varepsilon}{(di/dt)} = -\frac{0.20}{(-50)} = 4\text{mH}$

Illustration 6. An average emf of 20 V is induced in an inductor when the current in it is changed from 2.5 amp in one direction to the same value in the opposite direction in 0.1 s. The self–inductance of the inductor is

(A) zero (B) 200 mH (C) 400 mH (D) 600 mH

Solution: As $\varepsilon = -L\frac{di}{dt}$

$$\frac{di}{dt} = \frac{(-2.5)-(2.5)}{0.10} = -50\ \text{A/s}$$

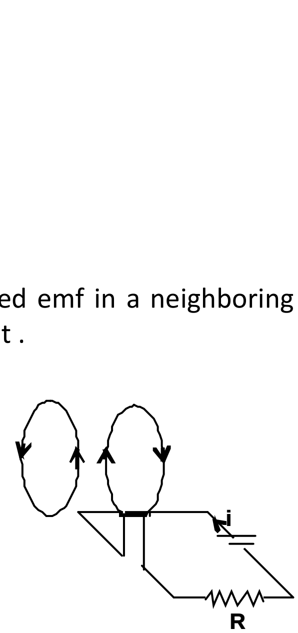
Self inductance $L = -\frac{\varepsilon}{(di/dt)} = -\frac{20}{50} = 400\text{m}$

Mutual Inductance

A changing current in one circuit causes a changing magnetic flux and an induced emf in a neighboring circuit. The net flux linked with the second circuit is proportional to current in first circuit .

i.e. $N_2\phi_2 = Mi_1$

The proportionality factor is called mutual inductance.



Also $M = \frac{N_2\phi_2}{i_1} = \frac{N_1\phi_1}{i_2}$

Or, $\varepsilon_2 = -M\frac{di_1}{dt}$ & $\varepsilon_1 = -M\frac{di_2}{dt}$

Note : Proceeding in the same way as in self inductance the mutual inductance M of solenoid of length ℓ and area of cross–section A and with number of turns N_{-1} and N_2 in primary and secondary coils is found to be $M = \dfrac{\mu N_1 N_2 A}{l}$.

Illustration 7. The coefficient of mutual induction between the primary and secondary of a transformer is 5 H. Calculate the induced emf in the secondary when 3 ampere current in the primary is cut off in 2.5×10^{-4} second.

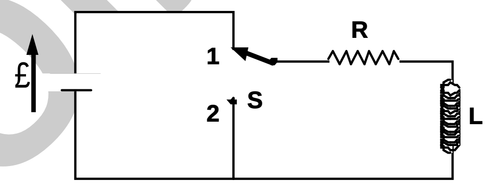
Solution: Induced emf in the secondary $\varepsilon_s = -M\dfrac{di_p}{dt} = -5\dfrac{3}{1/4000} = -6\times10^4\,V$

The negative sign merely indicates that the emf opposes the change.

R J L Circuit

Growth of current in an inductor

When current is allowed through an inductor the growing current induces an e.m.f. which opposes the growth of current in the inductor. When the switch is connected to the terminal 1, the current grows in the inductor.

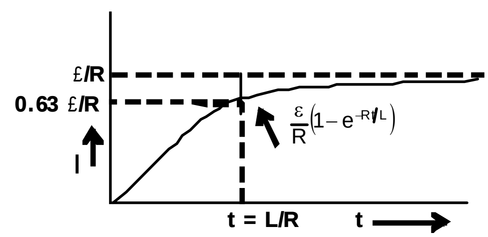


At position 1 of the switch, applying Kirchhoff's law to the closed circuit.

$$-IR - \frac{LdI}{dt} + \varepsilon = 0$$

Solving the differential equation, we get,

$$I = \frac{\varepsilon}{R}\left[1 - e^{\frac{-Rt}{L}}\right], \quad \frac{L}{R} = \tau$$



where τ is called the time constant of the circuit.

ALTERNATING CURRENT

Voltage and Currents in AC Circuits

Up to now we have considered voltage source and current in one direction only. In many cases we come across situations where the direction of current changes with time and the source provides voltage varying with time. One such case is when voltage and current vary like a sine function with time. These are called alternating voltage (a.c. voltage) and alternating current (a.c. current). Electricity supply provided at our homes and offices fall in this category. Main advantage of using this a.c. voltage and a.c. current is that a.c. voltage can easily be converted to lower or higher value by use of transformers and these can be economically transmitted over long distances.

Mean Value of Voltage and Current

The mean value of sinusoidal current or voltage in one complete cycle is zero. For half cycle, the mean value can be found as given below.

$$I = I_0 \sin \omega t;\; I_{mean} = \left[\frac{\int_0^{T/2} I\,dt}{\int_0^{T/2} dt}\right] = \frac{1}{T/2}\left[-\frac{I_0}{\omega}\cos\omega t\right]_0^{T/2}$$

$$= \frac{2}{T}\frac{I_0 T}{2\pi}\left[1-(-1)\right] = \frac{2I_0}{\pi}, \quad \text{Similarly,} \qquad v_{mean} = \frac{2V_o}{\pi}$$

Root Mean square value of voltage and current (V_{rms} and I_{rms})

$$I = I_0 \sin \omega t; \qquad I_{rms} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2}$$

$$I_{rms}^2 = \frac{1}{T}\int_0^T I_0^2 \sin^2 \omega t dt = \frac{I_0^2}{2T}\int_0^T (1-\cos 2\omega t)dt$$

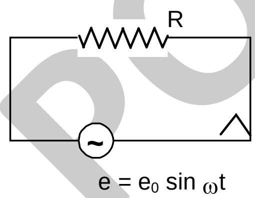
$$= \frac{I_0^2}{2T}\left[T - \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{I_0^2}{2}$$

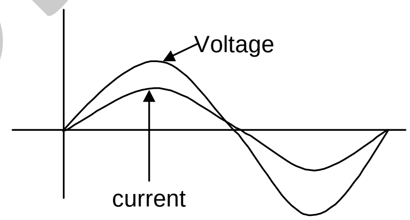
$$\therefore \qquad I_{rms} = \frac{I_0}{\sqrt{2}}, \qquad \text{similarly} \qquad V_{rms} = \frac{V_0}{\sqrt{2}}.$$

Impedance

In any circuit the ratio of the effective voltage to the effective current is defined as the impedance Z of the circuit. Its unit is ohm.

AC circuit with a resistor



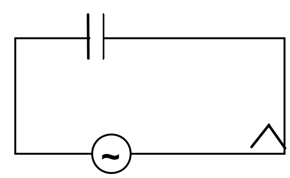


Instantaneous current

$$i = \frac{E}{R} = \frac{e_0}{R}\sin \omega t = i_0 \sin \omega t$$

where $i_0 = \frac{e_0}{R}$ = current amplitude. Thus the voltage and the current in an A.C. circuit containing pur**e** resistance are in phase.

AC circuit with a capacitor



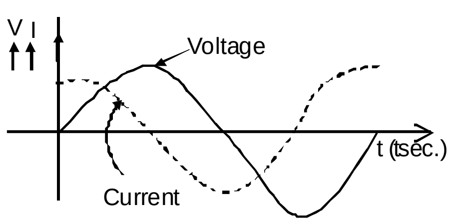
Instantaneous charge on the Capacitor

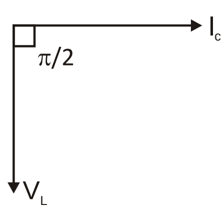
$Q = CV = Ce_0 \sin \omega t$, $I = \frac{dQ}{dt} = \omega Ce_0 \cos \omega t = i_0 \cos \omega t$

$= i_0 \sin(\omega t + \pi/2)$ where $i_0 = \omega Ce_0 = \frac{e_0}{1/\omega C} = \frac{e_0}{X_c}$

where $X_c = \frac{1}{\omega C}$ is known as capacitive reactance.

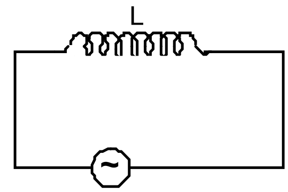
The following diagrams show graphical representation and phasor treatment of current and voltage illustrating the phase difference between them.





In capacitor, voltage lags the current or the current leads the voltage by $\pi/2$

AC circuit with an inductor



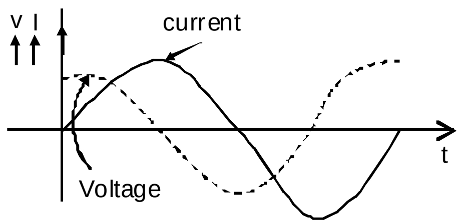
L = inductance in an ac circuit, $e = e_0 \sin(\omega t) = L\dfrac{di}{dt}$

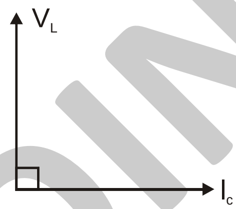
$$i = -\frac{e_0}{L_\omega}\cos(\omega t) + c = -i_0\cos(\omega t) + c$$

$= i_0 \sin(\omega t - \pi/2)$ where $i_0 = \dfrac{e_0}{\omega L} = \dfrac{e_0}{X_1}$

[∵ Voltage is sinusoidal so current should be also sinusoidal so c = 0)

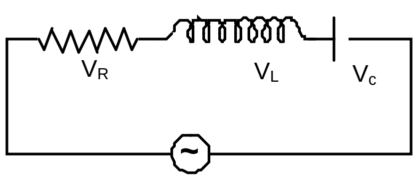
where X_L is known as inductive reactance

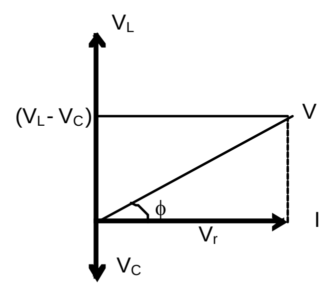




In an inductor voltage leads the current by $\pi/2$

LJCJR Circuit





$\vec{V} = \vec{V}_R + \vec{V}_L + \vec{V}_C$

$V = \sqrt{(V_L - V_C)^2 + V_R^2} = I\sqrt{(X_L - X_C)^2 + R^2}$,

where $X_L = \omega L$ & $X_C = \dfrac{1}{\omega C}$

⇒ $V = IZ$, where $Z = \sqrt{(X_L - X_C)^2 + R^2}$ is known as the impedance of the circuit.

Phase angle ϕ, $\tan\phi = \dfrac{V_L - V_C}{V_R} = \dfrac{X_L - X_C}{R}$

Let us study the phase relationship between current and e.m.f. in L.C.R. series circuit in the following case :

1. Where $\omega L > \dfrac{1}{\omega C}$, it follows that tan$\phi$ is positive, i.e. ϕ is positive. Hence, in such a case, voltage leads the current.

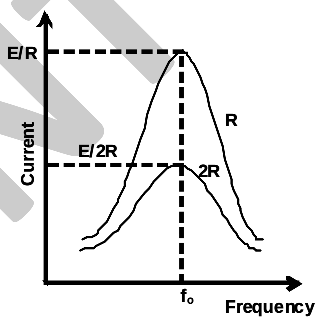
2. When $\omega L > \dfrac{1}{\omega C}$, it follows that tan$\phi$ is negative, i.e. ϕ is negative. Hence in such a case, voltage lags behind the current.

3. When $\omega L > \frac{1}{\omega C}$, it follows that tan$\phi$ is zero, i.e. ϕ is zero. Hence in such a case, current and voltage are in phase with each other.

4. In fact, when $\omega L > \frac{1}{\omega C}$, the impedance of the circuit would be just equal to R (minimum). In other words, the LCR–series circuit will behave as a purely resistive circuit. Due to the minimum value of impedance, the current in LCR–series circuit will be maximum. This condition is known as resonance.

$\Rightarrow \qquad \omega_0 = \frac{1}{\sqrt{LC}}$ where $\omega = \omega_o$ resonant frequency; $f_0 = \frac{1}{2\pi\sqrt{LC}}$

If R, L and C are constant, and the frequency f of the applied emf is raised continuously from zero, the peak current varies as shown in figure. At first the current is very small, increases to maximum when the frequency increase to the resonance value f_o, and then falls again.



It is interesting to note that before resonance the current leads the applied emf, at resonance it is in phase, and after resonance it lags behind the emf. Series resonant circuit is also called acceptor circuit.

Illustration 8. An LCR circuit has L = 20 mH, C = 50 μF and R = 10 Ω connected to a power source of 100 sin 400 t volt. Find out the rms value of current in the circuit.

Solution: $L = 20 \times 10^{-3}$ H, $C = 50 \times 10^{-6}$ F and R = 10 Ω, ω = 400 rad/sec.

Impedance

$$z = \sqrt{\left(400 \times 20 \times 10^{-3} - \frac{1}{400 \times 50 \times 10^{-6}}\right)^2 + 10^2} = 43.17\,\Omega$$

$$i_{rms} = \frac{100}{\sqrt{2}} \cdot \frac{1}{43.17} = 43.17\,\text{A} = 1.64\,\text{A}$$

POWER

In an a.c. circuit the instantaneous power is the product of the instantaneous value of the current and the voltage. $e = e_0 \sin \omega t$; ; $I = i_0 \sin(\omega t - \phi)$ $-90^0 \le \phi \le 90^0$

$P = e_0 i_0 \sin \omega t \sin(\omega t - \phi)$

$P_{av} = \frac{\int_0^T P dt}{\int_0^T dt} = \frac{1}{2} E_0 I_0 \cos\phi = E_{rms}\ I_{rms}\ \cos\phi$ = apparent power x cosϕ

where cos ϕ is known as power factor.

Power factor in L–C–R circuit is $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

If $\cos\phi = 0$, or $\phi = \pi/2$ $\qquad \Rightarrow \qquad P_{av} = 0$

i.e. when the current and the voltage differ in phase by 90°, Under this condition, the current is known as wattless current, because the average power consumed in the circuit is zero.

Illustration 9. A series LCR circuit containing a resistance of 120 Ω has angular resonance frequency 4×10^5 rad s^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. Find the values of L and C. At what frequency the current in the circuit lags the voltage by 45^0 ?

Solution : At resonance as X = 0, $I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2} A$ and $V_L = IX_L = I\omega L$,

$$L = \frac{V_L}{I_\omega} = \frac{40}{(1/2) \times 4 \times 10^5} = 2 \times 10^{-4} \text{ Henry}$$

at resonance, $\omega L = \frac{1}{\omega C}$ so $C = \frac{1}{\omega^2 L}$

i.e., $C = \frac{1}{0.2 \times 10^{-3} \times (4 \times 10^5)^2} = \frac{1}{32} \mu F$

Now in case of series LCR circuit, $\tan\text{¤} = \frac{X_L - X_C}{R}$

For the current to lag the applied voltage by 45^0

$$\tan 45^0 = \frac{\omega L - \frac{1}{\omega C}}{R}$$

i.e., $1 \times 120 = \omega \times 2 \times 10^{-4} - \frac{1}{\omega (1/32) \times 10^{-6}}$

i.e., $\omega^2 - 6 \times 10^5 = \omega - 16 \times 10^{10} = 0$

i.e., $\omega = \frac{6 \times 10^5 + 10 \times 10^5}{2} = 8 \times 10^5 \frac{rad}{s}$

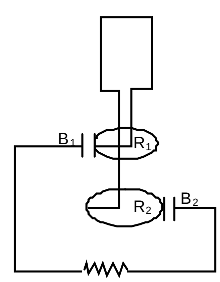
A.C. GENERATOR

An AC generator is a machine which converts mechanical energy to electrical energy based on electromagnetic induction.

Principle:

When the magnetic flux through a closed loop changes, it induces an emf in the coil. It lasts as long as the change in flux continues. A coil forming a closed loop is rotated in a magnetic field and change in flux due to rotation of coil produces induced emf.

Construction:



1. **Armature:** Armature is a rectangular coil. It consists of a large no. of turns of insulated copper wire wound over a soft iron core. The soft iron core is used to increase the magnetic flux.
2. **Field Magnet:** A strong electromagnet is provided with a magnetic field of the order of 1–2 tesla. It has concave north and south poles. The armature rotates between these poles.
3. **Slip Rings:** These are two hollow metallic rings. Ends of the armature coil are connected

to these two rings R_1 and R_2 separately. These provide moving contact hence these are called slip rings.

4. **Brushes:** Graphite brushes B_1 and B_2 keep contact with slip rings $R_1 \propto R_2$ and pass on current from the armature coil to the external resistance load.

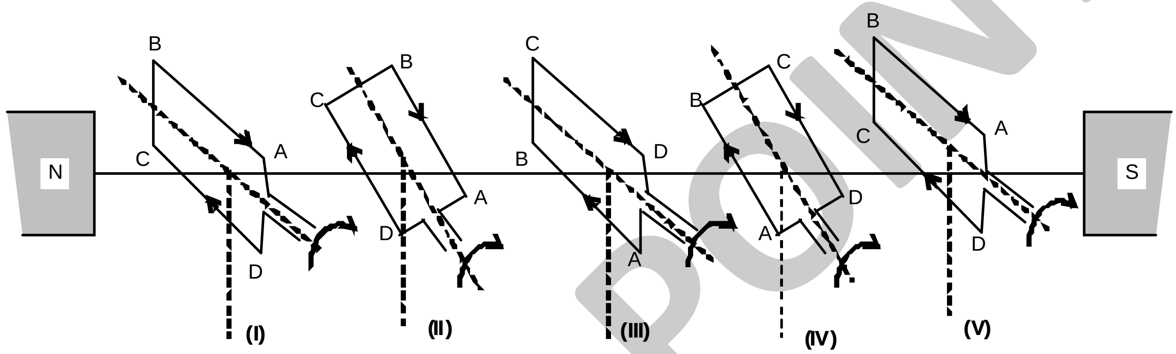
Working:

As the coil rotates between the N– S poles, magnetic flux linked with the coil changes. When normal to the coil area is parallel to the magnetic field lines, flux through it is maximum but rate of change of flux is minimum hence induced emf is zero.

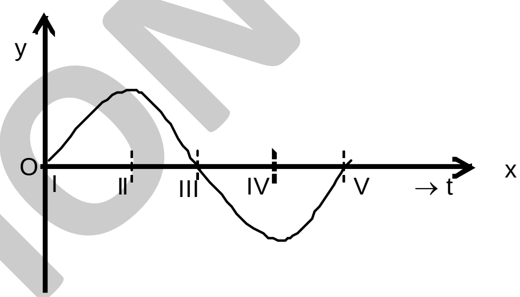
In the position where normal to the coil area is perpendicular to the magnetic field lines, rate of change of flux is maximum as coil wire move perpendicular the field lines. Hence at this position, induced emf is maximum.

The direction of induced emf can be determined by Fleming's right hand rule.

After half the rotation, the directions of current changes as the motion of arms are just opposite to the motion in first half. Applying Flemings right hand rule, the current direction is reversed, position III to V as compared to position I to III.



(Figure1: Variation of induced emf with time (I, II, III and IV are positions of coil as shown in the figure I)



If the magnetic field is B, the angular of the coil is ω (i.e. $\frac{d\theta}{dt} = \omega$), there are n no. of coils and area of coils is A, magnetic flux.

$\phi = nBA \cos \theta = nBA \cos\omega t$ $\quad\quad \therefore \frac{d\phi}{dt} = -nBA.\omega \sin \omega t$

Induced emf, $e = -\frac{d\phi}{dt}$

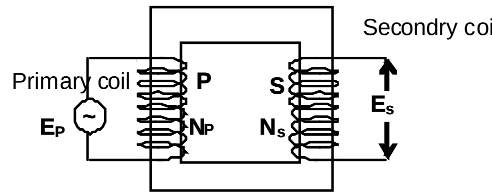
$e = nBA\omega \sin \omega t = e_0 \sin \omega t$

Here $e_0 = nBA\omega$

$e_{max} = nBA\,\omega$

Current $i = \frac{e}{R} = \frac{e_0}{R} \sin \omega t = i_0 \sin \omega t$ (here $i_0 = e_0/R$)

TRANSFORMER



A transformer is an electrical device used to convert AC current from low voltage & high current to high voltage and low current and vice–versa.

A transformer which increases the ac voltage is called step up transformer and which decreases the AC voltage is called step down transformer.

Principle:

A transformer works on the principle of mutual induction. When two coils are indirectely coupled, change in current or magnetic flux in one coil induces an emf in the other coil.

Construction:

Two coils Primary P and secondary S are wound around a soft iron core. A.C. input is applied to the primary coil and A.C. output is taken at the secondary coil. Laminated soft iron sheets are used in core to minimise eddy current and increase magnetic flux.

Step up transformers: Number of turns in secondary coil are more than those in the primary coil ($N_s > N_p$). It converts a low voltage high current to a high voltage low current.

Step down transformer: Number of turns in secondary coil are less than those in the primary coil. It converts a high voltage, low current to a low voltage, high current.

Working

When primary coil is connected to AC source, the magnetic flux linked with the primary changes. The magnetic flux of primary is passed through the secondary through the iron core. Therefore magnetic flux through secondary also changes. Due to the change in magnetic flux the induced emf is produced across the ends of secondary coil.

N_p = no of turns in primary
N_s = no of turns in secondary
E_p = voltage across primary
E_s = voltage across secondary
Since magnetic flux in directly proportional to no. of turns

$$\phi \propto N$$

$$\frac{\phi_s}{\phi_p} = \frac{N_s}{N_p}$$

$$\phi_s = \frac{N_s}{N_p} \times \phi_p$$

$$E_s = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{N_s}{N_p} \times \phi_p\right) = -\frac{N_s}{N_p}\left(\frac{d\phi_p}{dt}\right)$$

$$E_s = \frac{N_s}{N_p} \times E_p$$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

If $N_s < N_p$ then $E_s < E_p$, such type of transformer is called step down transformer.

Energy Losses in a Transformer:

Various types of losses are :

(a) **Flux losses:** – Flux of primary does not get 100 % linked up with secondary coil.

(b) **Copper losses:** – Energy lost as heat due to resistance in copper coils.

(c) **Iron Losses:** – Loss as heat in iron core due to eddy current losses.

(d) **Hysteresis losses:** Energy lost due to repeated magetization and demagnetization of iron core.

(e) **Humming losses:** Due to alternating current, the iron core vibrates producing humming noise. Some energy is lost in this.

Use of transformers:

Use of transformer are

1. Transmission of AC over long distances: Transmission losses are less when transmitted at very high voltage.
2. In induction furnaces – to heat metallic parts.
3. For welding where low voltage high current is required.
4. Vol tage regulators in electrical devices.

SUMMARY

- **Electromagnetic induction:** The production of electromotive force in a conductor when there is change in the magnetic flux linkage with a coil or when there is a relative motion of the conductor across a magnetic field.
- **Magnetic flux:** Magnetic flux through an area $d\vec{S}$ in a magnetic field $\vec{B}$ is $\phi_B = \int \vec{B}.d\vec{S}$
- **Faradays Law:** When the flux of a magnetic field through a loop changes, an emf is induced in the loop which is given by $\varepsilon = -\frac{d\phi}{dt}$

  where $\phi = \int \vec{B}.d\vec{S}$ is the flux through the loop.

This emf lasts as long as the magnetic field changes.

- **Lenz's law:** Lenz's law states that the polarity of the induced emf and the direction of the induced current is such that it opposes the change that has induced it.
- **Motional emf:** When a conducting rod of length ℓ moves with a constant velocity v in a magnetic field B such that B, v and ℓ are mutually perpendicular then the induced emf is $\varepsilon = B\ell v$

  This is called motional emf. If the circuit is completed the direction of the current can be worked out by Lenz's law.
- **Induced Electric field:** A time varying magnetic field induces an electric field. If a conductor is placed in this field, an induced emf is produced.

$$\varepsilon = \int \vec{E}\,.\,d\vec{l}$$

also, $$\varepsilon = -\frac{d\phi}{dt}$$

or, $$\int \vec{E}\,.\,d\vec{l} = -\frac{d\phi}{dt}$$

- **Eddy current:** Changing magnetic field induces currents in closed loops of irregular shapes in a conductor. These are called eddy currents. These dissipate thermal energy at the cost of kinetic energy, thus causing electromagnetic damping.
- **Self induction:** A current carrying loop produces a magnetic flux through the area

  $\phi = Li$ where L is self inductance of the loop.

  If current changes in the loop the induced emf ε is given by

  $$\varepsilon = -\frac{d\phi}{dt} = -L\frac{di}{dt}$$

  For a solenoid of n turns,

  $$\varepsilon = -n\frac{d}{dt}\int \vec{B}.d\vec{s}$$

  self inductance of a long solenoid is

  $L = \mu_r \mu_0 n^2 \ell A$

  where μ_r is relative magnetic permeability of the core, n is no. of turns per unit length, ℓ is the length of the solenoid and A is the cross–sectional area of the solenoid.
- **Mutual inductance:** A changing current in one circuit causes a changing flux and hence an induced emf in a neighboring circuit.

  $N_2\phi_2 = Mi_1$

  where M is mutual inductance of ciols $1 \propto 2$.

  $$M = \frac{N_2\phi_2}{i_1} = \frac{N_1\phi_1}{i_2}$$

  or, $$\varepsilon_2 = -M\frac{di_1}{dt}, \quad \varepsilon_1 = -M\frac{di_2}{dt}$$
- **Alternating voltage & Alternating current:** Voltage and current varying sinusoidally with time are called alternating voltage (Alternating voltage) and alternating current (Alternating current)

  $V = V_0 \sin\omega t$

  $I = I_0 \sin \omega t$
- **Mean values of voltage and current:**

  (a) In one complete cycle,

  $$\overline{V} = 0, \quad \bar{I} = 0$$

  (b) In half cycle,

  $$\overline{V} = \frac{2V_0}{\pi}, \quad \bar{I} = \frac{2I_0}{\pi}$$

  (c) Root mean square values

  $$V_{rms} = \frac{V_0}{\sqrt{2}}, \quad I_{rms} = \frac{I_0}{\sqrt{2}}$$ where v_0 and I_0 are the peak voltage and current.
- **AC circuit with a resistor:** $E = E_0 \sin \omega t = \frac{E_0}{R}\sin\omega t$

Resistance: R

- **AC circuit with a capacitor:** $V = E_0 \sin\omega t$

  $$I = \frac{E_0}{1/\omega C}\sin(\omega + \pi/2) \text{ (voltage lagging)}$$

  $$x_c = \frac{1}{\omega C}$$ is capacitive reactance.

- **AC circuit with an inductor:** $V = E_0 \sin \omega t$

  $I = I_0 \sin(\omega t - \pi/2)$ (voltage leading)

  $X_c = \omega L$ inductive reactance

- **LCR circuit:** Impedance, $z = \sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}$

  for phase angle ϕ, $\tan\phi = \frac{X_L - X_C}{R}$

- **Resonance:**

  $$\omega L = \frac{1}{\omega C}$$

  the impedance of the circuit is equal to R only.

  $$\omega_0 = \frac{1}{\sqrt{LC}}$$

  $$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{(resonant frequency)}$$

- **Power:**

  Average power, $P_{av} = E_{rms}\, I_{rms} \cos\phi = \frac{1}{2}E_0 I_0 \cos\phi$

  where $\cos\phi$ is power factor

  Power factor in LCR circuit,

  $$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}}$$

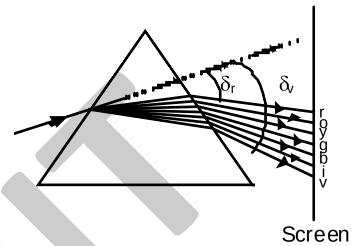
FINAL EXERCISE

1. What is electromagnetic induction? Explain Faraday's laws of electromagnetic induction.
2. State Lenz's law. Show that Lenz's law is a consequence of law of conservation of energy.
3. Distinguish between the self-inductance and mutual-inductance. On what factors do they depend?
4. How much e.m.f. will be induced in a 10H inductor in which the current changes from 10A to 7A in 9s?
5. Explain why the reactance of a capacitor decreases with increasing frequency, whereas the reactance of an inductor increases with increasing frequency?
6. What is impedance of an LCR series circuit? Derive an expression for power dissipated in a.c. LCR circuit.
7. A light bulb in series with an A.C. generator and the primary winding of a transformer glows dimly when the secondary leads are connected to a load, such as a resistor, the bulb in the primary winding will brighten, why?
8. If the terminals of a battery are connected to the primary winding of transformer, why will a steady potential differences not appear across the secondary windings.
9. The power supply for a picture tube in a colour television (TV) set typically requires 15,000V A.C. How can this potential difference be provided if only 230V are available at a household electric outlet?
10. Would two coils acts as transformer without an iron core? If so, why not omit the core to save money?
11. An ac source has a 10-volt out-put. A particular circuit requires only a 2V A.C. input. How would you accomplish this? Explain.
12. A person has a single transformer with 50 turns on one part of the core and 500 turns on the other. Is this a step-up or a step-down transformer? Explain.
13. Some transformers have various terminals or "taps" on the secondary so that connecting to different tap puts different functions of the total number of secondary windings into a circuit? What is the advantage of this?
14. A transformer in an electric welding machine draws 3A from a 240V A.C. power line and delivers 400A. What is the potential difference across the secondary of the transformer?
15. A 240-V, 400W electric mixer is connected to a 120-V power line through a transformer. What is the ratio of turns in the transformer? and How much current is drawn from the power line?
16. The primary of a step-up transformer having 125 turns is connected to a house lighting circuit of 220 Vac. If the secondary is to deliver 15,000 volts, how many turns must it have?
17. The secondary of a step-down transformer has 25 turns of wire and primary is connected to a 220V ac. line. If the secondary is to deliver 2.5 volt at the out-put terminals, how many turns should the primary have?

11 DISPERSION & SCATTERING OF LIGHT

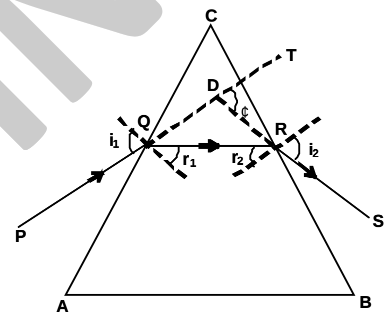
Dispersion

White light is actually a mixture of light of different colours. Therefore when white light gets refracted under certain conditions, its components bend by different amounts and separate out. This phenomenon of splitting of light into its component colours, due to different refractive indices for different colours, is called dispersion of light.

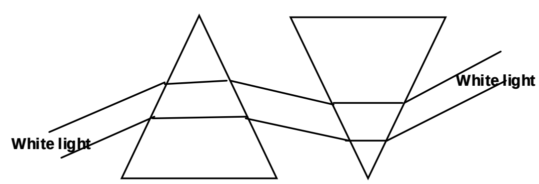


This phenomenon arises due to the fact that refractive index varies with wavelength. It has been observed for a prism that μ decreases with the increase of wavelength, i.e. $\mu_{blue} > \mu_{red}$.

If a narrow beam of light falls on one of the faces of glass prism, it emerges out from other face. We can observe that white light split into a band of different colours. In this band, one can distinguish seven prominent colours. These are Violet, Indigo, Blue, Green, Yellow, Orange and Red (acronym VIBGYOR). This band of colours is called spectrum. These colours can also be seen in a rainbow. We can observe that the violet light deviates most and the red light deviates the least. This indicates that the refractive index of glass is largest for violet and least for red.



We can also recombine these colours withthe help of another prism. Another prism can be kept inverted with respect to first. Then the final light obtained after the second prism is again a white light.



Angular dispersion, $\theta = \delta_v - \delta_r$

Dispersive power, Ratio of angular dispersion to mean deviation.

$$\omega = \frac{\delta_v - \delta_r}{\delta} \text{ where } \delta \text{ is deviation of mean ray (yellow)}$$

As $\delta_v = (\mu_v\ 1)A$, $\delta_r = (\mu_r - 1)A$,

$$\therefore \quad \omega = \frac{\mu_v - \mu_r}{\mu_y - 1}; \quad \text{where } \mu_y = \frac{\mu_v + \mu_r}{2}$$

Illustration 1. *Find the dispersion produced by a thin prism of 18^o having refractive index for red light = 1.56 and for violet light = 1.68.*

Solution: We know that dispersion produced by a thin prism

$\theta = (\mu_v - \mu_R)A$

Here $\mu_v = 1.68$, $\mu_R = 1.56$ and $A = 18^o$.

$\theta = (1.68 - 1.56) \times 18^o = 2.16^o$.

Average deviation without dispersion

This means an achromatic combination of two prisms in which net or resultant dispersion is zero and deviation is produced. For the two prisms,

$$(\mu_v - \mu_r)A + (\mu_v' - \mu_r')A' = 0$$

$$\Rightarrow \qquad A' = -\frac{(\mu_v - \mu_r)A}{(\mu'_v - \mu'_r)} \text{ and } \omega\delta + \omega'\delta' = 0$$

where ω and ω' are the dispersive powers of the two prisms and ¢ and ¢à their mean deviations.

Total deviation, $\delta_{net} = (\mu_y - 1)A\left(1 - \frac{\omega}{\omega'}\right)$.

Dispersion without Average Deviation

A combination of two prisms in which deviation produced for the mean ray by the first prism is equal and opposite to that produced by the second prism is called a direct vision prism. This combination produces dispersion without deviation.

For deviation to be zero, $(\delta + \delta') = 0$

$$\Rightarrow \qquad (\mu - 1)A + (\mu' - 1)A' = 0$$

$$\Rightarrow \qquad A' = -\frac{(\mu - 1)A}{(\mu' - 1)}$$ (–ve sign ⇒ prism *A*à has to be kept inverted)

Total angular dispersion, $\theta = (\mu_y - 1)A\,(\omega - \omega')$.

SCATTERING OF LIGHT

As sun light travels through the earth's atmosphere, it gets scattered by the large number of molecules present scattering represents basically change in the direction of light.

According to Rayleigh, Intensity of scattered light (I_s) varies inversely as the fourth power of the wavelength of incident light.

$$\text{i.e.} \qquad I_s \propto \frac{1}{\lambda_4}$$

These rays do not undergo any change in wavelength on scattering.

Examples:

(i) Blue colour of sky is due to scattering of sunlight. As the blue colour has a shorter wavelength than red, therefore blue colour is scattered much more strongly. Hence the sky looks blue.

(ii) At the time of sun rise and sunset, the sun is near the horizon. The rays from the sun have to travel a larger part of the atmosphere. As $\lambda_b < \lambda_r$ and intensity of scatted light $\propto \frac{1}{\lambda_4}$, therefore, most of the blue light is scattered away. Only red colour, which is least scatted enters our eyes and appears to come from the sun. Hence the sun looks red both at the time of sun rise and sun set.

SUMMARY

- Light of single wavelength or colour is said to be monochromatic but sunlight, which has several colours or wavelengths, is polychromatic.
- The splitting of light into its constituent wavelengths on entering an optically denser medium is called dispersion.
- A prism is used to produce dispersed light, which when taken on the screen, forms the spectrum.
- The angle of deviation is minimum if the angles of incidence and emergence become equal. In this situation, the beam is most intense for that colour.
- The rainbow is formed by dispersion of sunlight by raindrops at definite angles for each colour so that the condition of minimum deviation is satisfied.
- Rainbows are of two types : primary and secondary. The outer side of the primary rainbow is red but the inner side is violet. The remaining colours lie in between to follow the order (VIBGYOR). The scheme of colours gets reversed in the secondary rainbow.
- The blue colour of the sky, the white colour of clouds and the reddish colour of the Sun at sunrise and sunset are due to scattering of light.
- When light radiation undergoes scattering from a transparent substance, then frequency of scatered radiation may be greater or less than frequency of incident ratiation. This phenomenon is known as Raman effect.

FINAL EXERCISE

1. Would you prefer small-angled or a large-angled prism to produce dispersion. Why?

2. Under what condition is the deviation caused by a prism directly proportional to its refractive index?

3. Explain why the sea water appears blue at high seas.

4. Derive an expression for refractive index of a glass prism if a white light incident on prism and gets deviated.

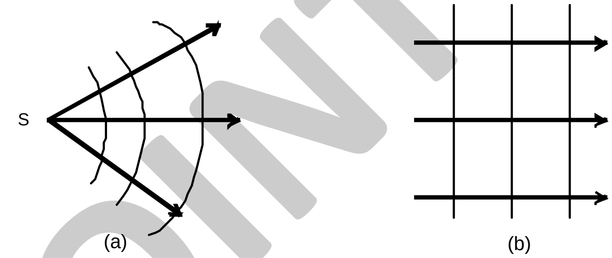
12 WAVE PHENOMENA & LIGHT

Wave front and wave normal:

A wave front can be defined as the locus of all the points of the medium to which the waves reach simultaneously, so that all the points vibrate in the same phase. If the distance of the source is small, the wave-front is spherical. For large distance, parallel beam of light gives rise to a plane wave front. A linear source, like an illuminated slit produces a cylindrical wavefront.

Wave normal:

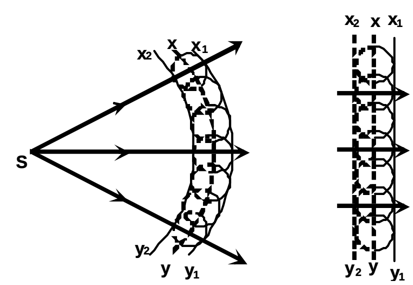
A perpendicular drawn to the surface of a wave-front at any point, in the direction of propagation of light, is called a wave normal. A wave front carries light energy in a direction perpendicular to its surface. This direction is represented by a wave normal. The direction in which light travels is also called a ray of light. Thus, a wave normal is same as a ray of light.



The successive positions of a spherical wave-front originating from a point source S and the corresponding wave normal are shown in fig. (a). It can be seen that the wave normal or rays are radial in the case of a spherical wave-front. In figure (b), the successive positions of a plane wave-front travel from left to right and the corresponding wave normal are shown. The wave normal or rays in this case are parallel to each other.

Huygens' Principle:

1. Every point on a wave-front acts as a secondary source of light and sends out secondary wavelets in all directions. They are effective only in the forward sense. The waves travel with the speed of light in the medium.
2. The position of the wave-front at a later instant is given by the surface of tangency or envelope of all the secondary wavelets at that instant. (An envelope is a curve tangential to a family of surfaces).



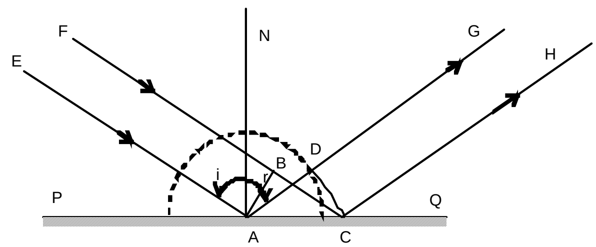
Refer to fig (a). XY is a portion of sphere of radius vt. Here v is the velocity of propagation of light wave. XY is called the primary wavefront.

Similar is the case in figure (b).

Reflection of a plane wave-front at a plane surface:

PQ is a plane reflecting surface (say a plane mirror). A plane wave-front AB bounded by two rays (wave-normals)

EA and FB is approaching PQ obliquely. When AB touches the surface PQ at A, then according to Huygens' principle, A acts like a secondary source and sends out secondary wavelets traveling in the same medium only. As time progresses different points on AB will come in contact with PQ and secondary wavelets will start from these points. If the disturbance at B reaches point C in time t, then the distance BC = Vt, where V is the speed of light in the medium.



In the same time, the secondary waves starting from A, travel the distance Vt. With centre A and radius BC = Vt. Draw a hemispherical surface (semi circle in two dimensions) and draw a tangent CD to this surface.

The points C and D are in the same phase. Hence, CD represents the reflected wave-front at the time t and it moves parallel to itself. Join AD, AG and CH are the reflected rays. Draw AN $\perp$ PQ.

In triangles ABC and ADC

(1) AC is common (2), AD = BC = Vt (by construction), (3) $\angle ABC = \angle ADC = 90^0$.

$\therefore$ The triangles are congruent $\therefore$ $\angle BAC = \angle ACD$.

From the figure,

$\angle EAN = i$, $\angle NAD = r$,

$\angle EAN = i = \angle BAC$ $\quad [\because \quad \angle EAN + \angle NAB = \angle NAB + \angle BAC = 90^0]$

$\angle NAD = r = \angle ACD.$ $\quad [\because \quad \angle NAD + \angle DAC = \angle DAC + \angle ACD = 90^0]$

$\therefore i = r$

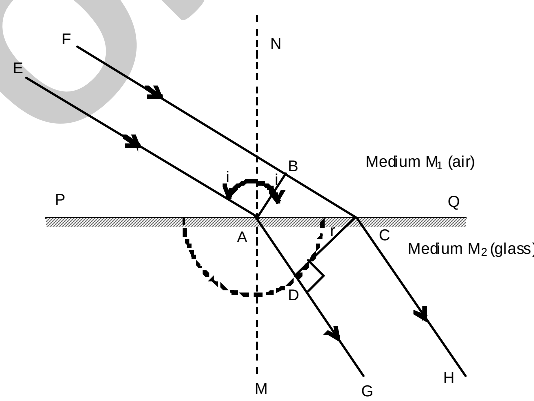
Thus, we get the following laws of reflection.

(i) The angle of incidence = angle of reflection.

(ii) Similarly, from the figure we find that the incident ray and the reflected ray lie on the opposite sides of the normal at the point of incidence and all three lie in the same plane (plane of the paper).

Thus, the laws of reflection are proved on the basis of Huygens' wave theory.

Refraction of a plane wave-front at a plane surface:



A plane wavefront AB bounded by the wave normals (rays) EA and FB is approaching obliquely the surface PQ. By Huygens' principle when the wavefront touches PQ at A, the point A becomes a secondary source and sends out secondary wavelets in all directions. But in the case of refraction of light, we consider the secondary wavelets traveling in the medium M_2 only. If the disturbance at B reaches C in time t then BC = V_1t. In the same time, secondary wavelets from A travel a distance V_2t in M_2.

With centre A and radius V_2t, draw a hemi-spherical surface (semicircle in two dimensios) in M_2. Through C, draw the tangent CD to this wavefront. In time t, different points on AB come in contact at various points between A and C and they become the secondary sources. CD is tangential to all the secondary wavelets emitted by the secondary sources and it represents the refracted wavefront. It moves parallel to itself.

Join A and D then AD = V_2t

Draw NAM PQ. From the figure

$\angle EAN = i = \angle BAC$

$\angle MAD = r = \angle ACD$

In ΔABC: $\sin i = \dfrac{BC}{AC}$

In ΔADC ; $\sin r = \dfrac{AD}{AC}$

$$\therefore \frac{\sin i}{\sin r} = \frac{BC/AC}{AD/AC}$$

$$\therefore \frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{V_1 t}{V_2 t} = \frac{V_1}{V_2}$$

Since V_1 and V_2 are constants in the two media, $\frac{\sin i}{\sin r}$ = constant

This is Snell's law and the ratio $\frac{\sin i}{\sin r}$ is known as the refractive index of the second medium (M_2) w.r.t. the first medium (M_1). It is denoted by ${}_1n_2$.

Thus, $${}_1n_2 = \frac{\sin i}{\sin r} = \frac{V_1}{V_2}$$

Thus Snell's law is proved.

Similarly, from the figure we find that the incident ray, the refracted ray and the normal to the refracting surface at the point of incidence lie in the same plane.

Note 1: By definition,

Absolute refractive index, $n = \frac{c}{V}$

Where c = speed of light (of a given frequency) in vacuum, and

V = speed of light (of the same frequency) in the medium.

$$n_1 = \frac{c}{V_1};\ n_2 = \frac{c}{V_2}$$

$${}_1n_2 = \frac{V_1}{V_2} = \frac{n_2}{n_1}$$

Note 2: Frequency (ν) of radiation is determined by the source. On refraction, frequency of radiation does not change, but speed and wavelength are changed.

Thus $V_1 = \nu\lambda_1$ (in medium 1)

And $V_2 = \nu\,\lambda_2$ (in medium 2)

$$\frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$

Thus, $${}_1n_2 = \frac{n_2}{n_1} = \frac{\sin i}{\sin r} = \frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$

For all practical purposes, speed of light in air (V_a) $\approx$ c.

Thus, the laws of refraction are proved.

Illustration 1. *Light is incident on a glass slab making an angle of 30^0 with the surface. If the speed of light in air is 3 ò 10^8 m/s and the refractive index of glass is 1.5, find:*

(i) the angle of refraction.

(ii) the speed of light in glass.

Solution: Given that,

$V_{air} = 3 \times 10^8$ m/s

$n_g = 1.5$

$i = 90 - 30 = 60^0$

(i) $\quad {}_a n_g = \dfrac{\sin i}{\sin r}$

$\Rightarrow \quad \sin r = \dfrac{\sin 60^0}{1.5} = \dfrac{\sqrt{3}/2}{3/2}$

$\Rightarrow \quad r = 35^0\ 16$

(ii) $\quad {}_a n_g = \dfrac{V_{air}}{V_{glass}}$

or, $\quad V_{glass} = \dfrac{V_{air}}{n_g} = 2 \times 10^8\ m/s$.

Interference: The phenomenon of interference of light is based on the principle of superposition of waves, which is stated as follows: "When two or more waves arrive at a point simultaneously, each wave produces its own displacement or effect at that point which does not depend upon other waves." The resultant displacement at that point is the vector sum of the instantaneous displacements due to individual waves meeting at that point. The resultant displacement is maximum, if the displacements due to the waves are in the same phase.

The resultant displacement is minimum, if the two displacements are in opposite phase. The resultant displacement is zero if the amplitudes of the two waves are equal.

Therefore, the phenomenon of enhancement or cancellation of displacement produced due to the superposition of waves is called interference.

Condition for steady interference pattern:

(i) The two sources of light must be coherent.

(ii) The two sources of light must be monochromatic.

(iii) The two sources must be equally bright.

(iv) The sources should be narrow.

(v) The two sources should be close to each other.

Note:

- When two waves with amplitudes A_1 and A_2 superimpose at a point, the amplitude of resultant wave is given by $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$
  where ϕ is the phase difference between the two waves at that point.
- Intensity $\propto A^2$. Hence, for I to be constant, ϕ must be constant.
- Intensity $(I) = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$.
- When ϕ changes with time arbitrarily at a point, the intensity = $I_1 + I_2$.
- When ϕ does not change with time, we get an intensity pattern and the sources are said to be coherent. Coherent sources have a constant phase relationship in time.
- The intensity at a point becomes a maximum when $\phi = 2n\pi$ and there is constructive interference, where n = 0, 1, 2, ...
- If $\phi = (2n - 1)\pi$ there is destructive interference. (Hence n is a non-negative integer)

Illustration 2. *Four incoherent waves are expressed as $y_{-1} = a_1 \sin \mu t$, $y_2 = a_2 \sin 2\mu t$, $y_3 = a_3 \cos \mu t$ and $y_4 = a_4 \sin[\mu t + (\pi/3)]$. In which two waves, interference is possible.*

Solution: Out of all these four waves, no two can produce interference pattern. Here, the sources are not coherent.

Illustration 3. *If instead of using a single source of light, we use two different sources*

(A) interference pattern will never be obtained

(B) broad fringe width will be obtained

(C) very thin fringe pattern will be obtained

(D) overlapped fringes will be obtained

Solution : **(A)** Interference pattern is obtained only if coherent sources of light are used, i.e. two sources having a constant phase difference, which is produced when two slits get light from a single source. Two different sources can never have a constant phase difference.

Coherent sources

Two sources are said to be coherent if they emit light waves of the same frequency having the same wavelengths and always maintained with constant phase difference.

For experiments, two virtual sources are formed from a single source which can act as coherent sources.

Conditions for obtaining two coherent sources of light:

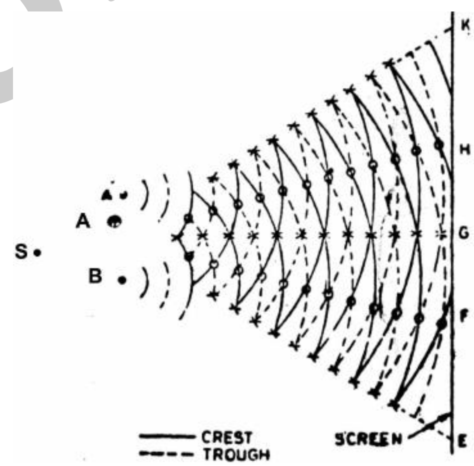
(i) Coherent sources of light should be obtained from a single source by same device.

(ii) The two sources should give monochromatic light.

(iii) The path difference between light waves from two sources should be small.

Young's double experiment

In the year 1802, Young demonstrated the experiment on the interference of light. He allowed sunlight to fall on a pinhole S and then at some distance away on two pinholes A and B.



A and B are equidistant from S and are close to each other. Spherical waves spread out from S. Spherical waves also spread out from A and B. These waves are of the same amplitude and wavelength. On the screen interference bands are produced which are alternatively dark and bright. The points such as E are bright because the crest due to one wave coincides with the crest due to the other and therefore they reinforce each other. The points, such as F are dark because the crest of one falls on the trough of the other and they neutralise the effect of each other.

Phase difference (a¤) and path difference (a¤)

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

If the path difference between the two waves is λ, the phase difference $= 2\pi$ and time difference is T. For a path difference x, the phase difference is δ

Then, $\delta = \dfrac{2\pi x}{\lambda}$

Determination of phase difference

The phase difference between two waves at a point will depends upon:

(a) the difference in path lengths of the two waves from their respective sources;

(b) the refractive index of the medium;

(c) initial phase difference, between the sources, if any;

(d) reflections, if any, in the path followed by the waves.

➢ In the case of light waves, the phase difference on account of path difference

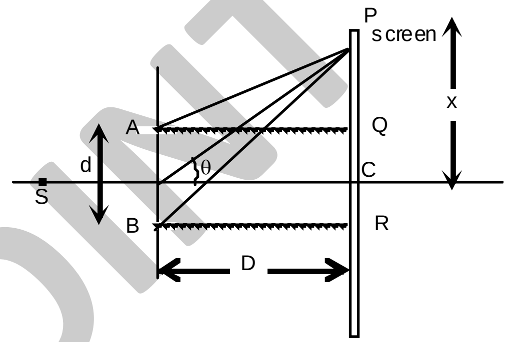
$$= \frac{\text{Optical path difference}}{\lambda} 2\pi, \quad \text{where } \lambda \text{ is the wavelength in free space.}$$

$$= \frac{\mu[\text{Geometrical path difference}]}{\lambda} 2\pi$$

In the case of reflection, the reflected disturbance differs in phase by π with respect to the incident one if the wave is incident on a denser medium from a rarer medium. No such change of phase occurs when the wave is reflected in going from a denser medium to a rarer medium.

Analytical treatment of Interference:

Assume a monochromatic source of light S emits waves of wavelength λ and two narrow pinholes A and B are located at equidistant from S. A and B act as virtual coherent sources. Let the amplitude of waves be a. The phase difference between the two waves reaching the point P, at any instant, is δ.



If y_1 and y_2 are the displacements

$y_1 = a \sin \omega t$

$y_2 = a \sin (\omega t + \delta)$

$\therefore \quad y = y_1 + y_2 = a \sin \omega t + a \sin (\omega t + \delta)$

$= a \sin \omega t + a \sin \omega t \cos\delta + a \cos \omega t \sin \delta$

$= a \sin \omega t (1 + \cos \delta) + a \cos \omega t \sin \delta$

Taking $a(1+ \cos \delta) = R \cos \theta$ and $a \sin \delta = R \sin \theta$

$\Rightarrow \quad y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$ and

$= R \sin (\omega t + \theta)$

This represents the equation of simple harmonic vibration of amplitude R.

Next $\quad R^2 \sin^2\theta + R^2 \cos^2\theta = a^2 \sin^2\delta + a^2 (1 + \cos\delta)^2$

$\Rightarrow \quad R^2 = a^2 \sin^2\delta + a^2 + a^2 \cos^2\delta + 2a^2 \cos\delta$

$\Rightarrow \quad R^2 = 2a^2 + 2a^2 \cos\delta$

$= 4a^2 \cos^2 (\delta/2)$

intensity at a point is given by the square of the amplitude

$\Rightarrow \quad I = R^2$

$I = 4a^2 \cos^2 \delta/2$

Special cases:

(i) When the phase difference $\delta = 0, 2\pi, 2(2\pi) \ldots\ldots n(2\pi)$

or $\quad x = 0, \lambda, 2\lambda, \ldots\ldots n\lambda$

then $\quad I = 4a^2$

(ii) When the phase difference $\delta = \pi, 3\pi \ldots\ldots (2n + 1)\pi$

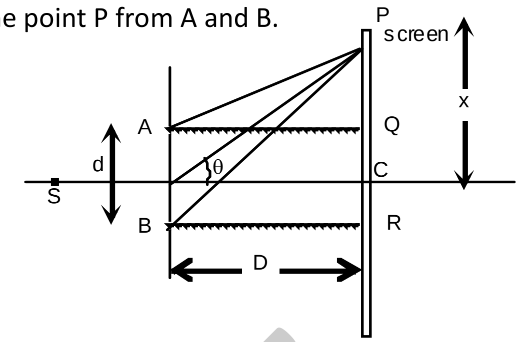
or $\quad x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots\ldots (2n + 1)\lambda/2$

then $\quad I = 0$

Fringe Width

Consider a narrow coherent source S and pinholes A and B, equidistant from S. A and B act as two coherent sources separated by a distance d. Let a screen the placed at a distance D from the coherent source. The point C on the screen is equidistant from points A and B, thus, the point C has maximum intensity.

Consider a point P at a distance x from C. The waves reach at the point P from A and B.



Here $PQ = x - d/2$

$PR = x + d/2$

$$(BP)^2 - (AP)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$\Rightarrow$ $(BP)^2 - (AP)^2 = 2xd$

$\Rightarrow$ $BP - AP = \dfrac{2xd}{BP + AP}$

But $BP \approx AP \approx D$

$\therefore$ Path difference $BP - AP = \dfrac{2xd}{2D} = \dfrac{xd}{D}$

Phase difference $= \dfrac{2\pi}{\lambda}\left(\dfrac{xd}{D}\right)$

(i) Bright fringe:

$$\frac{xd}{D} = n\lambda \qquad \text{where } n = 0, 1, 2, 3, \ldots\ldots$$

$$x = \frac{n\lambda D}{d}$$

This equation gives the distance of the bright fringe from the point C.

When $n = 1$, $x_1 = \dfrac{\lambda D}{d}$

When $n = 2$, $x_2 = \dfrac{2\lambda D}{d}$

When $n = n$, $x_n = \dfrac{n\lambda D}{d}$

Therefore, the distance between any two consecutive bright fringes is

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d}$$

$\Rightarrow$ $\beta = \dfrac{\lambda D}{d}$ (which is fringe width.)

(ii) **Dark fringes :**

$$\frac{xd}{D} = (2n-1)\frac{\lambda}{2}, \qquad \text{where } n = 1, 2, 3 \ldots.$$

or $x = \dfrac{(2n-1)\lambda D}{2d}$

where, n = 1, $x_1 = \frac{\lambda D}{2d}$

n = 2 $x_2 = \frac{3}{2}\frac{\lambda D}{d}$

n = 3 $x_3 = \frac{5}{2}\frac{\lambda D}{d}$

The distance between any two consecutive dark fringes,

$$x_3 - x_3 = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d} \Longrightarrow x_2 - x_1$$

Thus, $\beta = \frac{\lambda D}{d}$

Which is same for bright fringe also.

Illustration 4. *A monochromatic light of wavelength 5100 $\overset{0}{A}$ from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 100 cm away is 1 cm, find the slit separation.*

Solution: $\beta = \frac{\lambda D}{d}$

$\lambda = 5100 \times 10^{-8}$ cm

D = 100 cm

Given, 10 β = 1 cm , β = 0.1 cm

$$d = \frac{\lambda D}{\beta} = \frac{5100 \times 10^{-8} \times 100}{0.1} = 0.051\,\text{cm}$$

Illustration 5. *In Young's experiment, two slits are 0.2 mm apart. The interference fringes for light of wavelength 6000 are formed on a screen 80 cm away from slits.*

(a) How far is the second bright fringe from the central fringe?

(b) How far is the second dark fringe from the central fringe?

Solution: Here, $d = 0.2 \text{ mm} = 2 \times 10^{-4}$ m

$\lambda = 6000\text{Å} = 6 \times 10^{-7}$ m

D = 80cm = 0.8m

(a) x = ?, n = 2 (for second bright fringe)

$$x = \frac{n\lambda D}{d} = \frac{2 \times 6 \times 10^{-7}}{2 \times 10^{-4}} = 4.8 \times 10^{-3}\text{m.}$$

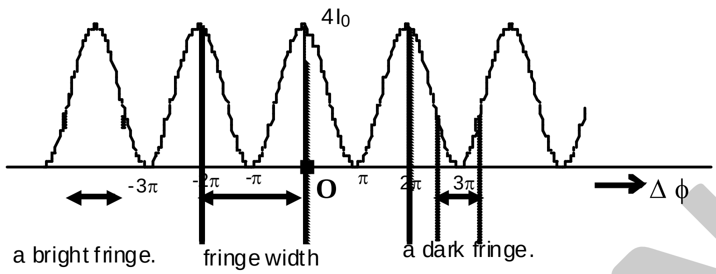
(b) x = ? n = 2 (for 2nd dark fringe)

$$x = (2n-1)\left(\frac{\lambda}{2}\right)\frac{D}{d} = \frac{3 \times 6 \times 10^{-7} \times 0.8}{2 \times 2 \times 10^{-4}} = 3.6 \times 10^{-3}\text{m.}$$

Intensity variation on screen

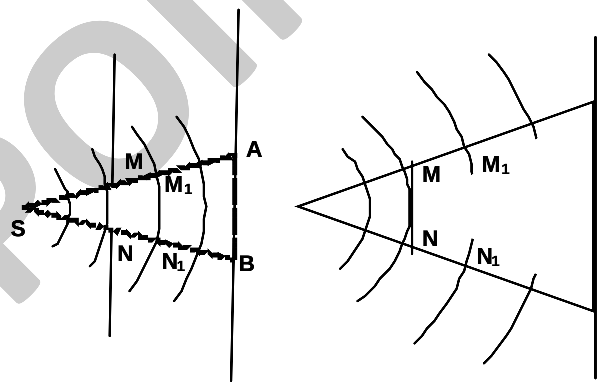
If A and I_o represent amplitude and intensity of each wave, then the resultant intensity at a point on the screen corresponding to the angular position θ as in figure below, is given by

$$I = 4I_0 \cos^2 \frac{\phi}{2} \qquad \text{where } \phi = \frac{2\pi(d\sin\theta)}{\lambda}$$



DIFFRACTION

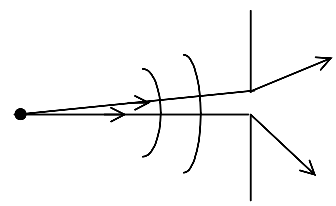
It is a matter of common experience that the path of light entering a dark room through a hole illuminated by sunlight is straight. This phenomenon of straight line motion can be explained by Newton's corpuscular theory. But it has been observed that when a beam of light passed through a small opening, it spreads to some extent into the region of geometrical shadow also. If the light energy is propagated in the form of waves, then similar to sound waves one would expect bending of a beam of light round the edges of an opaque obstacle or illumination of the geometrical shadow.



(a) Diffraction is the bending or spreading of waves that encounter an object (a barrier or an opening) in their path.

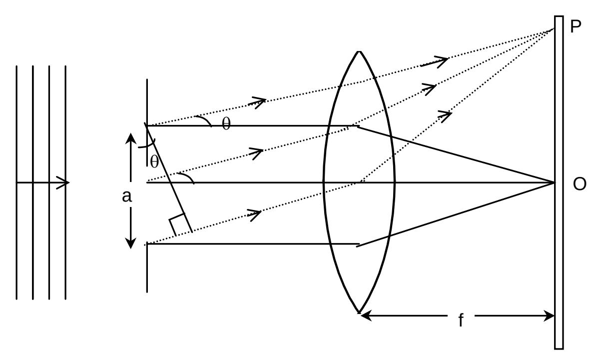
(b) In Fresnel class of diffraction, the source and/or screen are at a finite distance from the aperture.

(c) In Fraunhoffer class of diffraction, the source and screen are at infinite distance from the diffracting aperture. Fraunhoffer is a special case of Fresnel diffraction.



Single Slit Fraunhoffer Diffraction

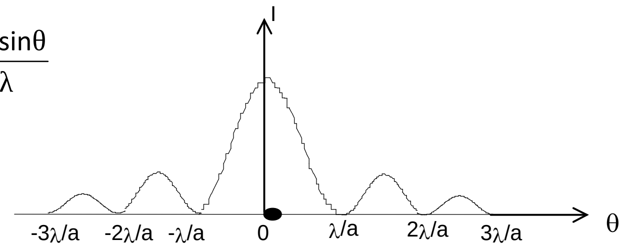
In order to find the intensity at point P on the screen as shown in the figure the slit of width 'a' is divided into N parallel strips of width Δx. Each strip then acts as a radiator of Huygen's wavelets and produces a characteristic wave disturbance at P, whose position on the screen for a particular arrangement of apparatus can be described by the angle θ.



The amplitudes ΔE_o of the wave disturbances at P from the various strips may be taken as equal if $\theta$is not too large.

The intensity is proportional to the square of the amplitude. If I_m represents the intensity at O, its value at P is

$$I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{where} \quad \alpha = \frac{\phi}{2} = \frac{\pi a \sin\theta}{\lambda}$$



A minimum occurs when, sin α = 0 and

$\alpha \neq 0$, so $\alpha = n\pi$, n = 1, 2, 3...

$$\Rightarrow \quad \frac{\pi a \sin\theta}{\lambda} = n\pi \quad \Rightarrow \quad a\sin\theta = n\lambda$$

Angular width of central maxima of diffraction pattern = $2\theta_1 = 2\sin^{-1}(\lambda/a)$

[θ_1 gives the angular position of first minima]

The concept of diffraction is also useful in deciding the resolving power of optical instruments.

Illustration 6. *Light of wavelength 6×10^{-5}cm falls on a screen at a distance of 100 cm from a narrow slit. Find the width of the slit if the first minima lies 1mm on either side of the central maximum.*

Solution: Here n = 1, $\lambda = 6 \times 10^{-5}$ cm.

Distance of screen from slit = 100 cm.

Distance of first minimum from central maxima = 0.1 cm.

$$\sin\theta = \frac{\text{Distance of } 1^{st} \text{ minima from the central maxima}}{\text{distance of the screen from the slit}}$$

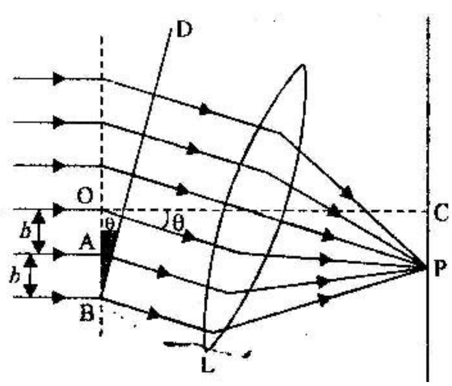
$$\theta_1 = \frac{0.1}{100} = \frac{1}{1000}$$

We know that a sinθ = nλ

$$a = \frac{\lambda}{\theta_1} = 0.06\,\text{cm}.$$

The diffraction grating:

It is glass plate upon which is ruled a large number of equally spaced opaque lines. These lines are generally several thousand per centimeter.



Consider the parallel rays making an angle θ with the normal (OC) to the grating. The rays are brought to focus on the screen at point P by a converging lens L.

If the ray AP travels a distance λ farther than ray BP, then waves from A and B will interfere constructively at P. The wave front BD makes an angle θ with the grating.

From the right angled triangle as shown in figure

$$\sin\theta = \frac{\lambda}{AB}$$

or, λ = AB sin θ

or, λ = b sin θ

This is the condition for reinforcement in the direction θ.

There will be other directions on each side of OC for which waves from adjacent slits differ in phase by 2λ, 3λ ....

In general, the grating equation may be written as

$b \sin \theta_n = n\lambda$

where b is the grating space and n is the order of the spectrum.

Rayleigh Criterion

According to Rayleigh criterion, when the central maximum in the diffraction pattern of one point source falls over the first minimum in the diffraction pattern of the other point source. Then the two point sources are said to have been resolved by the optical instrument.

Resolving power of microscope

The resolving power of microscope is its ability to form separate images of two point objects lying close together.

The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just revolved when seen through the microscope.

$$\therefore \quad \text{Resolving power of microscope} = \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda} = \frac{1}{\Delta d} = \frac{2u \sin\theta}{\lambda}$$

Where

(i) μ is the refractive index of the medium between the object and the objective of microscope.

(ii) λ is the wavelength of the light.

(iii) The angle θ subtended by the radius of the objective on one of the objects.

Resolving power of a telescope

The resolving power of a telescope is the reciprocal of the smallest angular separation between two distinct objects whose images are separated in the telescope. This is given by

$$d\theta = \frac{1.22\lambda}{a}$$

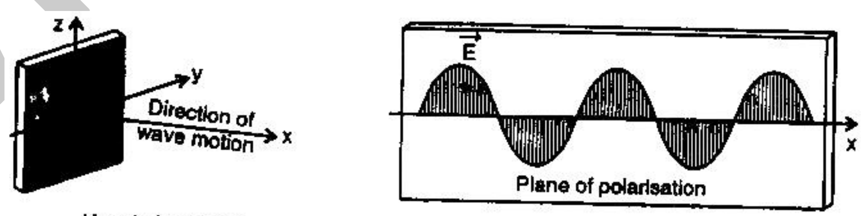
Where $d\theta$ is the angle subtended by the point object at the objective. λ is the wavelength of light used and a is the diameter of the telescope objective.

Clearly, a telescope having larger aperture objective gives a high resolving power.

Polarization

An ordinary beam of light consist of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$. Since, all directions of vibrations of $\vec{E}$ are equally probably therefore resultant electromagnetic wave is called un-polarized light and it is symmetrical about the direction of wave propagation as shown in figure (A).

However, if by some means we confine the vibrations of electric vector in one direction perpendicular to the direction of wave motion, the light is said to be plane polarized or linearly polarized as shown in the figure (B).

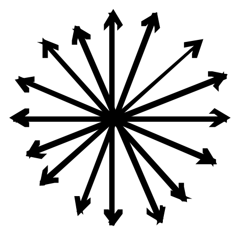


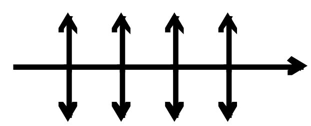
Unpolarised light (A) Plane polarised light (B)

Hence the phenomenon of confining the vibrations of a wave in a specific direction perpendicular to the direction of wave motion is called polarization. The plane perpendicular to the plane of polarization i.e. the plane in which no vibration occur is known as plane of vibration. The plane of polarization is that plane in which no vibrations occur.

For the shape of convenient representation, the vibrations may be assumed to be resolved into two rectangular components, in the planes of the paper and perpendicular to the plane of paper.

The vibrations in the plane of paper are represented by double arrow straight lines. While the vibrations perpendicular to the plane of paper are represented by dots as shown in figure.

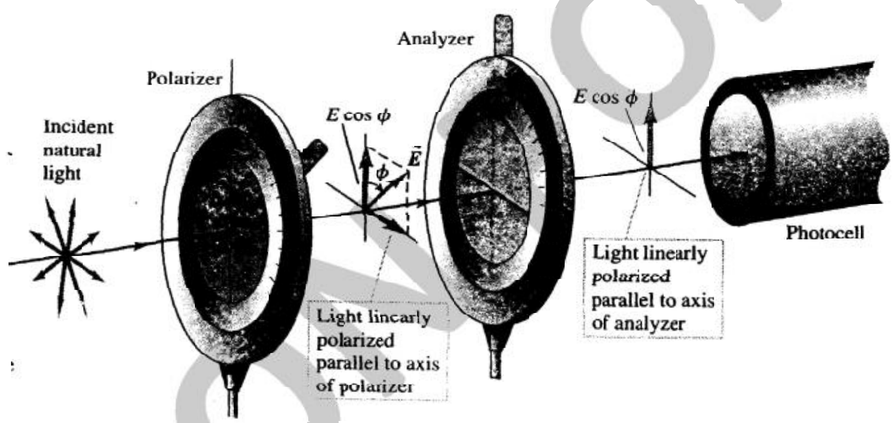




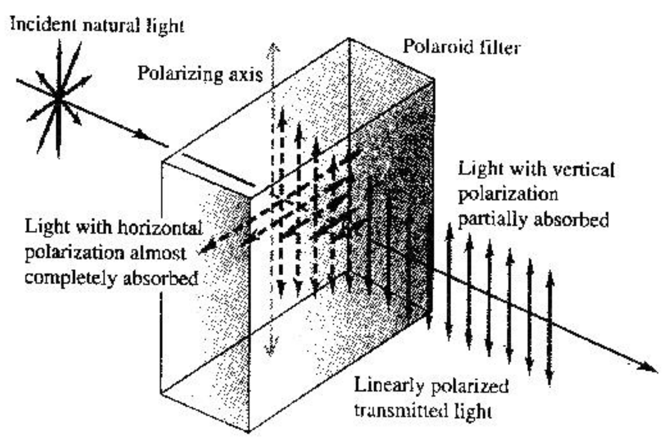
Representation of natural light

Natural light

Polarizing filter: The emitted light is a random mixture of waves linearly polarized in all possible transverse directions. Such light is called unpolarised light or natural light. To create polarized light from unpolarised natural light requires a filter that is analogous to the slot for mechanical waves.



The most common polarizing filter for visible light is a material known by the trade name Polaroid, widely used for sunglasses and polarizing filter for camera lenses.



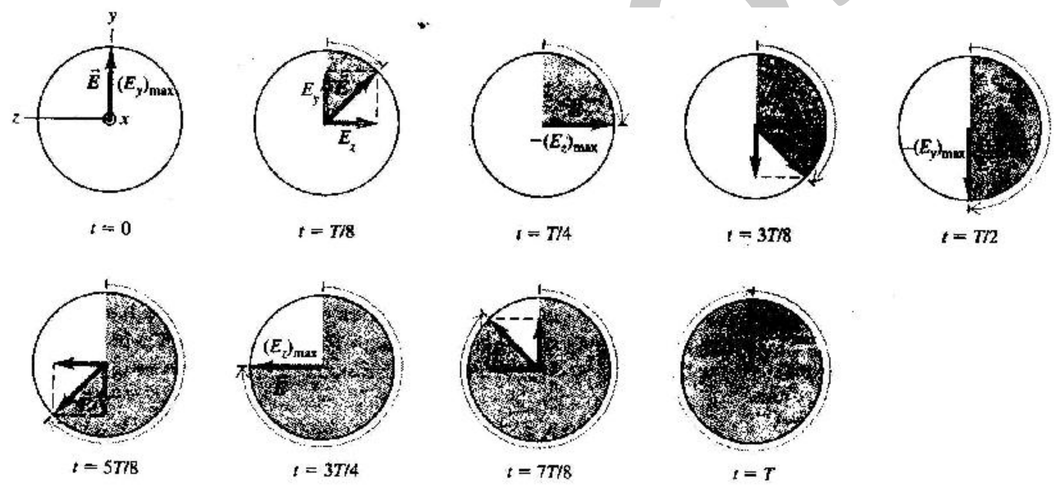
This material incorporates substances that have dichroism, a selective absorption in which one of the polarized component is absorbed much more strongly that the other. A Polaroid filter transmits 80 % or more of the intensity of a wave that is polarized parallel to a certain axis in the material, called the polarizing axis, but only 1 % or less for waves that are polarized perpendicular to the axis. In one type of Polaroid filter, long – chain molecules within the filter are oriented with their axis perpendicular to the polarizing axis. These molecules preferentially absort light that is polarized along their lengths.

An ideal polarizing filter passes 100 % of the incident light that it polarized in the direction of filter's polarizing axis but completely blocks all light that is polarized perpendicular to this axis.

When unpolarised light is incident on an ideal polarizer, the intensity of the transmitted light is exactly half that of the incident unpolarised light, no matter how the polarizing axis is oriented. Here's why we can resolve the $\vec{E}$ field of the incident wave into a component parallel to the polarizing axis and a component perpendicular to it. Because the incident light is a random mixture of all sates of polarization, these two components are, on average, equal.

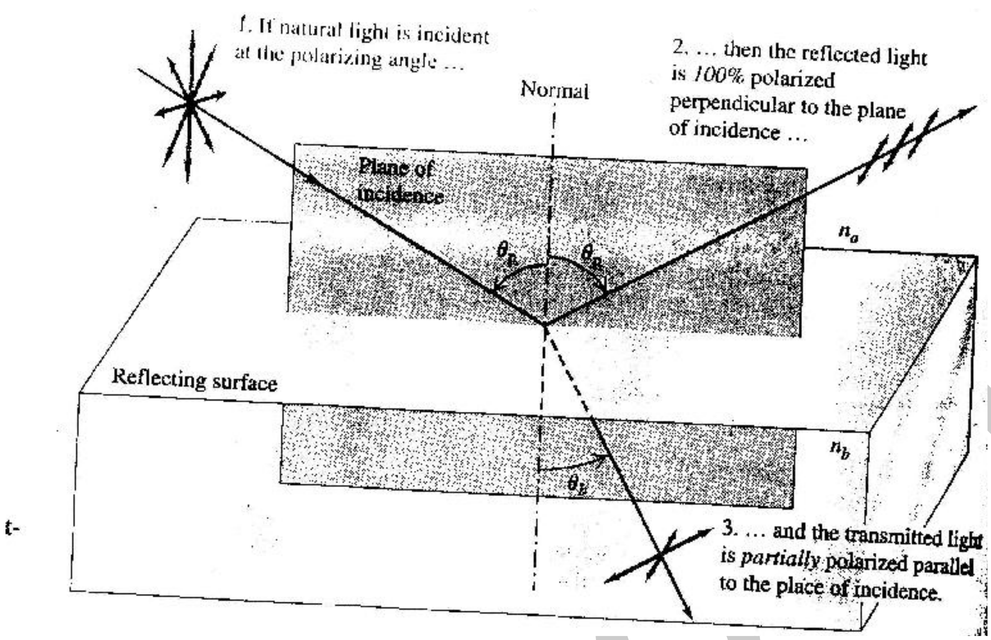
What happens when the linearly polarized light emerging from a polarizer passes through a second polarizer. Consider the general case in which the polarizing axis of the second polarizer or analyzer makes an angle ϕ with the polarizing axis of first polarizer.

Then $\quad I = I_{max} \cos^2 \phi$



Since the intensity of an electromagnetic wave is proportional to the square of the amplitude of the wave, the ratio of transmitted to incident amplitude is cos ϕ, so the ratio of transmitted to incident intensity is $\cos^2 \phi$

Polarization by reflection: Unpolarized light can be polarized, partially or totally, by reflection. When unpolarised natural light is incident on a reflecting surface between two transparent optical materials, then for the most of the incident, waves, for which the electric – field vector $\vec{E}$ is perpendicular to the plane of incidence are refracted more strongly than those for which $\vec{E}$ lies in this plane. In this case the reflected light is practically polarized in the direction perpendicular to the plane of incidence. But at one particular angle of incidence called the polarizing angle θ_p, the light for which $\vec{E}$ lies in the plane of incidence is not reflected at all but is completely refracted. At this same angle of incidence the light for which $\vec{E}$ is perpendicular to the plane of incidence is partially reflected and partially reflected and partially refracted. The refracted light is therefore completely polarized perpendicular to the plane of incidence.



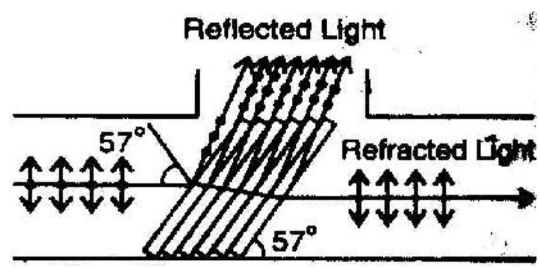
If natural light is incident at the polarizing angle.

Brewster's law: When the angle of incidence is equal to the polarizing angle i_p the reflected ray and refracted ray are perpendicular to each other. In this case the angle of refraction r_c becomes the complement of i_p,

so, $r_c = 90^0 - i_p$

$\Rightarrow \mu_1 \sin i_p = \mu_2 \sin (90^0 - i_p) \Rightarrow \tan i_p = \mu_2 / \mu_1$

Polarization by Refraction



By refraction method, a pile of glass is formed by taking 20 to 30 microscope slides and light is made to be incident at polarizing angle (57^0). In accordance with Brewster law, the reflected light will be plane polarized with vibrations perpendicular to plane of incidence and the transmitted light will be partially polarized. Since in one reflection about 15 % of the light with vibration perpendicular to plane of paper is reflected, therefore after passing through a number of plates as shown in figure emerging light will become plane polarized with vibrations in the plane of paper.

Double Refraction: When a ray light is refracted by a crystal of calcite it gives two refracted rays. This phenomenon is called double refraction.

When a ray of light AB is incident on the calcite crystal making an angle of incidence = i, it is refracted along two paths inside the crystal as shown in figure.

(i) along BC making an angle of refraction = r_2 and

(ii) along BD making an angle of refraction = r_1. These two rays emerge out along DO and CE which are parallel. The ordinary ray (O-ray) has a refractive index $\mu_0 = \dfrac{\sin i}{\sin r_1}$ and the extraordinary ray (e – ray) has a refractive index $\mu_0 = \dfrac{\sin i}{\sin r_2}$.

The e-ray and the e-ray travel with the same speed along a particular direction inside the crystal called the optic axis.

Dichroism

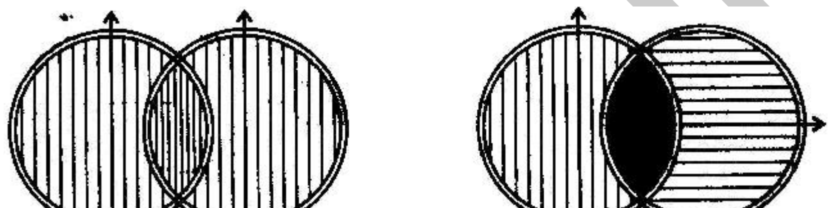
There are certain crystals and minerals which are doubly refracting and have the property of absorbing the ordinary and extraordinary rays unequally. In this way, plane polarized light is produced. The crystals showing this property are said to be dichroic and the phenomenon is known as dichroism. Tourmaline is a dichroic crystal and absorbs the ordinary ray completely. But these crystals are not stable and are affected by slightly strain. To remove this difficulty, a polarizer in the forms of large sheets is developed which is called Polaroid.

Polorids

Herapathite crystals are embedded in a volatile viscous medium and the crystals are aligned with their optics axes parallel. The layer of crystals are mounted between glass sheets so that the crystals are not spoilt.

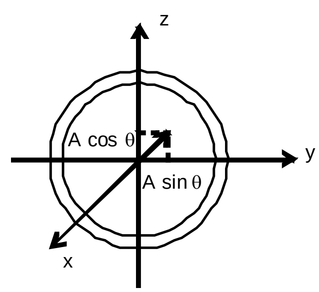
It is a sheet of polariser and also known as Polaroid.

Two Polaroid films mounted separately in rings between thin glass plates are used on the parallel position as shown in figure (a) light vibrating in the plane indicated by parallel lines is transmitted. In the crossed position as shown in figure (b). the axes of the polaroids are perpendicular to each other. So no light is transmitted.



Parallel-polaroids (A) Crossed-polaroids (B)

Intensity of light emerging from a Polaroid



If a plane polarized light of light intensity I_0 (=kA^2) is incident on a polaroid and its vibrations of amplitude A make an angle θ with the transmission axis, then the component of vibrations parallel to transmission axis, will be A $\cos\theta$ while perpendicular to it will be A $\sin\theta$

Now, Polaroid will pass only those vibrations which are parallel to its transmission axis.

i.e. A $\cos\theta$, so the intensity of emergent light will be

$$I = k(A\cos\theta)^2 = kA^2\cos^2\theta$$

$$I = I_0\cos^2\theta \qquad [\because\ I_0 = kA^2]$$

This law is called Malus law.

Illustration 7. *Two polarizing sheets are placed with their planes parallel, so that light intensity transmitted is maximum. Through what angle must either sheet be turned so that light intensity drops to half the maximum value ?*

Solution: According to law of Malus

$$I = I_0\cos^2\theta$$

$$\therefore\ \cos^2\theta = \frac{I}{I_0} = \frac{1}{2}$$

$$\therefore\ \cos\theta = \pm\frac{1}{\sqrt{2}}$$

$\Rightarrow$ $\theta = \pm 45^0$ or $\pm 135^0$

The effect will be same when any of the two sheets is turned through θ in any direction.

Uses of Polaroids

Polaroids are widely used as polarizing sun glass. Polaroid films are used to produce three – dimensional moving pictures. They are also used to eliminate the head light glare in motor cars.

SUMMARY

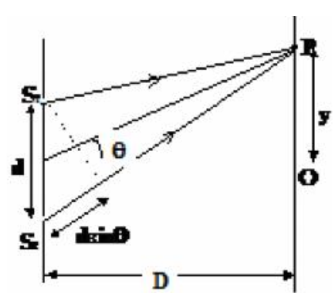
- If two coherent waves with intensity I_1 and I_2 are superimposed with a phase difference of ϕ, the resulting wave intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2}$ cos¤
- When sources are in phase:
  For maxima o.p.d. = $n\lambda$ (o.p.d. = Optical Path Difference)

  For minima o.p.d. = $\left(n - \frac{1}{2}\right)\lambda$

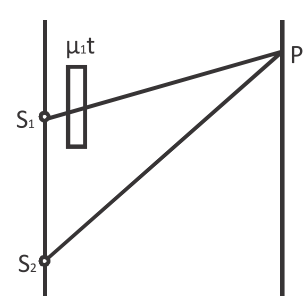
  Phase difference $\phi = \frac{2\pi}{\lambda}$ (o.p.d.)
- The phase difference between two waves at a point will depend upon
  (a) the difference in path lengths of the two waves from their respective sources.
  (b) the refractive index of the medium
  (c) initial phase difference, if any.
  (d) Reflections, if any, in the path followed by waves.
  In the case of light waves, the phase difference on account of path difference

  $$= \frac{\text{optical path difference}}{\lambda} 2\pi$$ where λ is the wavelength in free space.

  $$= \frac{\mu\,[\text{Geometrical path difference}]}{\lambda} 2\pi$$
- In the case of reflection, the reflected disturbance differs in phase by π with the incident one if the incidence occurs in rarer medium. There would be no phase difference if incidence occurs in denser medium.



- Young's Double Slit Experiment
  (i) For maxima $d\sin\theta = n\lambda$
  (ii) For minima $d\sin\theta = (2n + 1)\lambda/2$
  (iii) Fringe width , $\omega = \lambda D/D$
  (iv) Displacement of fringe pattern
- When a film of thickness 't' and refractive index 'μ' is introduced in the path of one of the source's light, then fringe shift occurs as the optical path difference changes.



  Optical path difference at
  P $= S_2P - [S_1P + \mu t - t]$
  $= S_2P - S_1P - (\mu - 1)t = y.d/D - (\mu - 1)t$

  $\Rightarrow$ n^{th} fringe shifted by $\Delta y = \frac{\Delta(\mu - 1)\,t}{d}$. As $\omega = D\lambda/d$, also $\Delta y = \frac{\omega}{\lambda}(\mu - 1)\,t$

FINAL EXERCISE

1. What is a wavefront? What is the direction of a beam of light with respect to the associated wavefront? State the Huygens' principle and explain the propagation of light waves.

2. Obtain the laws of reflection on the basis of Huygens' wave theory.

3. What is the principle of superposition of waves? Explain the interference of light.

4. Describe Young's double slit experiment to produce interference. Deduce an expression for the width of the interference fringes.

5. What would happen to the interference pattern obtained in the Young's double slit experiment when

(i) one of the slits is closed;

(ii) the experiment is performed in water instead of air;

(iii) the source of yellow light is used in place of the green light source;

(iv) the separation between the two slits is gradually increased; (v) white light is used in place of a monochromatic light;

(vi) the separation between the slits and the screen is increased;

(vii) two slits are slightly moved closer; and

(viii) each slit width is increased.

6. Distinguish between the polarized and unpolarized lights.

7. State and explain Brewster's law.

8. The polarising angle for a medium is 60^o. Calculate the refractive index.

9. For a material of refractive index 1.42, calculate the polarising angle for a beam of unpolarised light incident on it.

10. With the help of Huygens' construction, explain the phenomenon of diffraction.

13 STRUCTURE OF ATOM

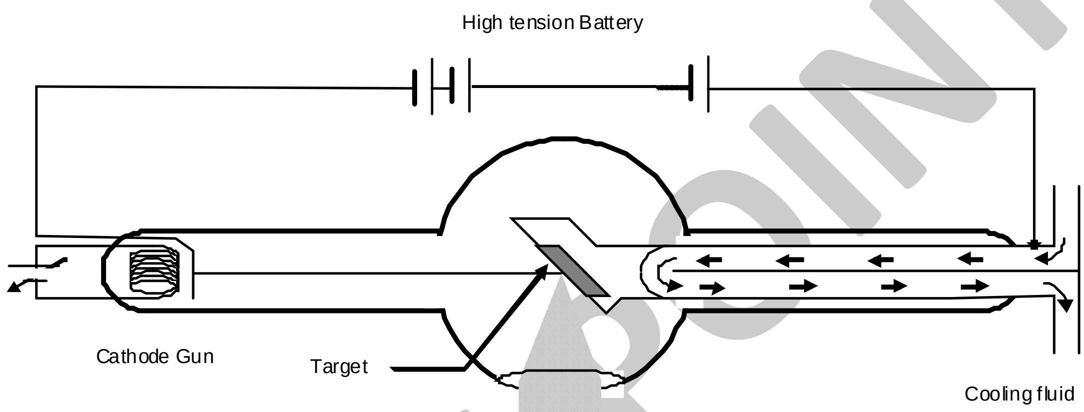
X-Rays

Characteristic and production of x-rays

(i) It is also known as inverse photoelectric effect, as energetic electrons produce electro-magnetic radiations.

(ii) Their wavelength is of the order of 1 A^o.

(iii) They are produced with an apparatus called Coolidge tube.



In a Coolidge tube, electrons are emitted by thermionic emission, accelerated across a very high potential difference V and made to hit a target. X-rays are produced and emerge out of a window. Water is circulated in the target to keep it cool.

Continuous and characteristic x-rays

(i) When an accelerated electron hits the target, the electron loses its energy in two processes. One process gives rise to continuous X-rays and the other process gives rise to characteristic X-rays.

(ii) When the electron loses its kinetic energy in several collisions with the atoms, a fraction of the lost kinetic energy is converted into electromagnetic radiations. This fraction can range from zero to one.

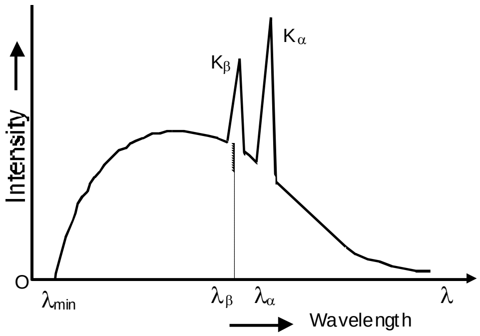
(iii) Corresponding to the maximum kinetic energy lost by an electron, we have the largest frequency of the X-ray or the shortest wavelength.

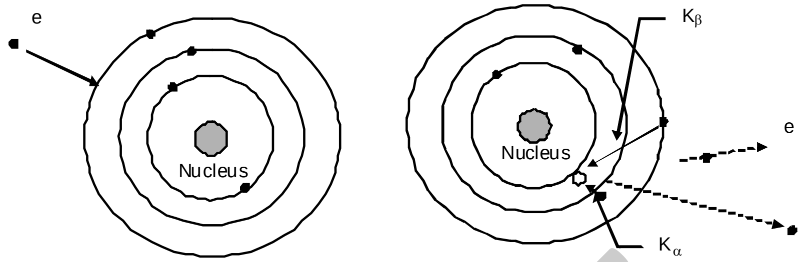
$$\frac{hc}{\lambda_{min}} = K_{max.} = eV \qquad \Rightarrow \qquad \lambda_{min} = \frac{hc}{eV}$$

The emitted wavelengths range from a minimum value all the way to infinity. Thus the name continuous spectrum.

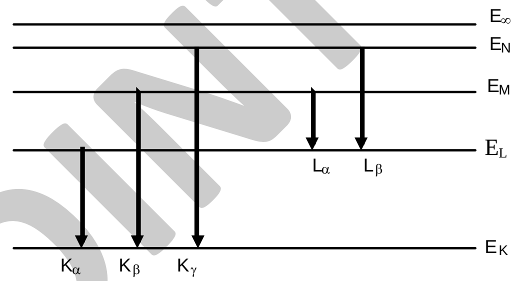
(iv) The other possibility is that the accelerated electron might knock out the inner electron of the target atom, whereby a vacancy is created in the inner orbit. Electrons from the higher orbit jump in to fill this vacancy, while releasing the energy difference as electro-magnetic radiations.

These wavelengths are characteristic of the material from which they are emitted. Hence the name Characteristic Spectra.





(v) To discuss energy transitions in X-rays, we take the ground state atom with all electrons intact as the zero-level. The atom with a vacancy in the K-shell is the one with the highest energy, as a lot of energy is required to dismiss a K-shell electron. Therefore we have the following energy diagram.



$$\lambda = \frac{hc}{E_K - E_L} \text{ for } K_\alpha \text{ wavelength.}$$

$$\lambda = \frac{hc}{E_L - E_M} \text{ for } L_\alpha \text{ wavelength and so on.}$$

Moseley's law

Moseley gave an empirical law.

$$\sqrt{f} = a(Z - b)$$

Where, f = frequency of X-ray,

Z = atomic number of target atom, a, b are constants.

It was proposed before the Bohr's theory, and we see that it agrees with the Bohr's model.

For K_α-line, the L-electron jumps to a vacancy in the K-level. So for the L electron, there are Z protons in the nucleus and an electron in the K-shell which screens of the positive charge. So the net charge the L electron experiences can be taken as (Z - 1)e.

Now, $$E = Rhc(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]; \qquad hf = Rhc\,(Z-1)^2 \cdot \left[\frac{3}{4}\right]$$

$$\sqrt{f} = \sqrt{\frac{3}{4}.Rc}\,(Z-1)$$

If we compare the above with Moseley's formula, we see that for K_α line, $a = \sqrt{\frac{3}{4}Rc}$ and b = 1.

Properties of x-rays

(i) X-rays do not get deflected by electric or magnetic fields.

(ii) When passed through gases, they produce ionisation.

(iii) X-rays diffraction is used to study crystal structures.

(iv) They have high penetrating power and are used in radiographs.

Alpha Particle Scattering Experiment:

1. Ernest Rutherford, in a series of experiments between 1906 and 1911 obtained valuable information on the structure of atom.

   In the alpha particle scattering experiment carried out by Rutherford, a beam of alpha particles was bombarded on a thin foil of gold. The observations were:

   1. Most of the α - particles passed through undeviated or with minor deviation.

   2. Some of the α - particles were deflected through large angles. Some of them (1 in 8000) were deviated by more than 90^0. They were turned back.

Based on the above observations, Rutherford concluded that

- atom contains positively charged tiny particles at the centre called nucleus of the atom.
- negatively charged particles electrons move around nucleus.
- space between nucleus and electrons is empty and this determines the size of an atom.

Bohr's theory of hydrogen like atoms

Consider an electron of charge -e and mass m, orbiting, with a speed v, a central hydrogen nucleus of charge +e. In classical electromagnetism, charges undergoing acceleration emit radiation and, therefore, they would lose energy. Therefore the electron should spiral towards the nucleus and the atom should collapse. In order to overcome this difficulty, Bohr suggested that in those orbits where the angular momentum is a multiple of $\frac{h}{2\pi}$, the energy is constant. Twelve years later, de Broglie proposed that a particle such as an electron may be considered to behave as wave of wavelength $\lambda = \frac{h}{p}$.

If electron jumps from a higher energy shell to another shell, it emits energy in the form of radiation, which is equal to energy difference of the two shells.

If the electron can behave as a wave, it must be possible to fit a whole number of wavelengths in an orbit. In this case a standing wave pattern is set up and the energy in the wave is confined to the atom. A progressive wave would imply that the electron is moving from the atom and is not in a stationary orbit.

If there are n waves in the circular orbit and λ is the wavelength.

$$n\lambda = 2\pi r$$

$$\therefore \quad \lambda = \frac{2\pi r}{n} = \frac{h}{mv} \quad \Rightarrow \quad mvr = \frac{h}{2\pi}(n)$$

The centripetal force is provided by the electrostatic attraction.

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \quad \therefore \quad r = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$$

Velocity of the electron in the nth orbit

$$v = \frac{e^2}{2\varepsilon_0 hn}$$

Kinetic energy of the electron in the nth orbit

$$K = \frac{1}{2}mv^2 = \frac{me^2}{8\varepsilon_0{}^2 h^2 n^2}$$

Potential energy of the electron

$$V = -\frac{me^4}{4\varepsilon_0^2 h^2 n^2}$$

$\therefore$ Total energy of the electron

$$E = K + V = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}$$

For hydrogen like-atoms with nuclear charge +Ze, the following expression holds true :

(i) Radius of the nth orbit, $r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2 Z} = r_0 \frac{n^2}{Z}$

r_o = radius of the hydrogen atom = 0.53 Å (n = 1)

(ii) Velocity of the electron in the nth orbit, $v_n = \frac{Ze^2}{2\varepsilon_0 hn} = v_0 \frac{Z}{n}$

(iii) Energy of the electron in the nth shell, $E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^2 n^2} = E_0 \frac{Z^2}{n^2}$

E_o = Energy of the electron (n = 1) in hydrogen atom, E_o- = -13.6 eV

(iv) Kinetic energy of an electron = – Total energy $= -\frac{1}{2} \times$ potential energy

Illustration 1. *A single electron orbits a stationary nucleus of charge +Ze, where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second orbit to third Bohr orbit. Find*

(a) the value of Z

(b) the energy required to excite the electron from the third to the fourth Bohr orbit.

(c)the wavelength of electromagnetic radiation required to remove the electron from first Bohr orbit to infinity.

(d) Find the K.E. , P.E. and angular momentum of electron in the 1st Bohr orbit.

(e) the radius of the first Bohr orbit.

[The ionisation energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3 10^{-11} m, velocity of light = 3 ò 10^8 m/s, Planck's constant = 6.6 ò 10^{-34} J.s.]

Solution: The energy required to excite the electron from n_1 to n_2 orbit revolving round the nucleus with charge +Ze is given by

$$E_{n_2} - E_{n_1} = \frac{Z^2 me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$E_{n_2} - E_{n_1} = Z^2 \times 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \text{electron volt.}$$

(a) Since 47.2 eV energy is required to excite the electron from n_1 =2 to n_2=3 orbit.

Therefore, $47.2 = z^2 \times 13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$$Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 25 \quad \text{or} \quad Z = 5.$$

(b) The energy required to excite the electron from n_1 = 3 to n_2 = 4 orbit is given by

$$E_4 - E_3 = 25 \times 13.6 \times \left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{25 \times 13.6 \times 7}{144} = 16.53\,\text{eV}$$

(c) The energy required to remove the electron from the first Bohr orbit to infinity (∞) is given by

$$E_\infty - E_1 = 13.6 \times Z^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 13.6 \times 25\,\text{eV}$$

In order to calculate the wavelength of radiation, we use frequency relation

$$E = \frac{hc}{\lambda} = 13.6 \times 25 \times (1.6 \times 10^{-19})\,\text{J}$$

or $$\lambda = \frac{\left(6.6 \times 10^{-34}\right) \times \left(3 \times 10^{8}\right)\text{m}}{13.6 \times 25 \times \left(1.6 \times 10^{-19}\right)} = 36.5\ \text{Å} = 340\ \text{eV}$$

(d) $$\text{K.E.} = \frac{1}{2}mv_1^2 = \frac{1}{2} \times \frac{Ze^2}{4\pi\varepsilon_0 r_1} = 340 \times 1.6 \times 10^{-19}\,\text{J}$$

P.E. = -2 × K.E. = -1086.8×10^{-19} J

Angular momentum = $mv_1r_1 = h/2\pi = 1.06 \times 10^{-34}$ Js

(e) The radius r_1 of the first Bohr's orbit is given by

$$r_1 = \frac{\varepsilon_0 h^2}{\pi m e^2}\frac{1}{Z} = \frac{0.53 \times 10^{-10}}{5} \qquad \left(\because \quad \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53 \times 10^{-10}\right)$$

$= 0.106 \times 10^{-10}$ m = 0.106 Å

Illustration 2. *Hydrogen atom in its ground state is excited by means of mono chromatic radiation of wavelength 975 Å. How many different lines are possible in the resulting spectrum. Calculate the longest wavelength among them. You can assume the ionization energy for hydrogen atom as 13.6eV.*

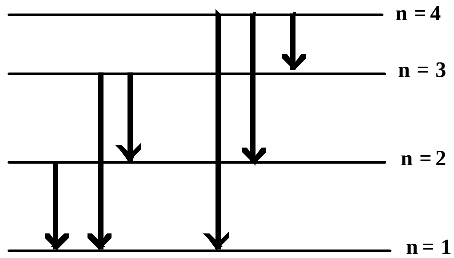
Solution: $\therefore$ Ionization energy of hydrogen atom = 13.6eV

$$\therefore\ 13.6\left(\frac{1}{1^2} - \frac{1}{n^2}\right) = 12.75 \qquad \left(\because \quad E = \frac{12400}{975}\text{eV} = 12.75\,\text{eV}\right)$$

$$\Rightarrow \frac{1}{n^2} = 1 - \frac{12.75}{13.6} = \frac{0.85}{13.6} = \frac{1}{16}$$

$$\therefore \frac{1}{n} = \frac{1}{4} \Rightarrow \qquad n = 4$$

$\therefore$ No. of lines possible in the resulting spectrum = ${}^4C_2 = 6$



The longest wavelength will be emitted for transition from 4th orbit to 3rd orbit with an energy

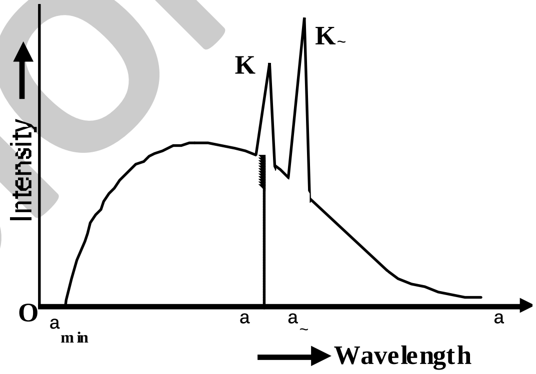
$$\Delta E = (-13.6\text{eV})\left(\frac{1}{4^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{7}{144}\text{eV}$$

$\therefore$ longest wavelength $\lambda = \frac{hc}{\Delta E} = 18800\text{Å}$

SUMMARY

- **X-Rays**
  When highly energetic electrons are made to strike on a metallic target, electromagnetic radiation called X-rays are emitted.

Continuous and Characteristic X-Rays



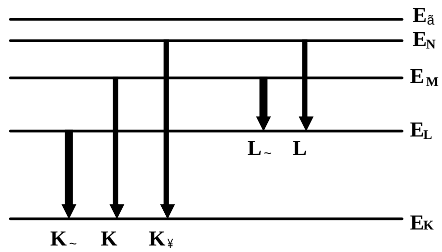
- **Continuous X-Rays**
  When the electron loses its kinetic energy during several collisions with the other atoms, a fraction of the lost kinetic energy is converted into electromagnetic radiations. This fraction can range from 0 to 1. Corresponding to the maximum kinetic energy lost by an electron, we have the largest frequency of the X-ray or the shortest wavelength.

$$\frac{hc}{\lambda_{min}} = K_{max} = eV$$

$$\Rightarrow \qquad \lambda_{min} = \frac{hc}{eV}$$

The emitted wavelengths ranges from a minimum value all the way to infinity. Thus, the name continuous spectrum.

Characteristic X-Rays



The accelerated electron might knock out the inner electron of the target atom, whereby a vacancy is created in the inner orbit. Electrons from the higher orbit jump in to fill this vacancy, while releasing the energy difference as electromagnetic radiations, these wavelengths are characteristic of the material from which they are emitted. Hence, the name Characteristic Spectra.

- ***Bohr's Theory***

This theory is applicable for hydrogen and similar atoms.

- **Bohr's theory for hydrogen-like atoms**

$$mvr = \frac{nh}{2\pi} \qquad \text{and} \qquad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} = \frac{Ze.e}{r^2}$$

$$\Rightarrow \qquad r_n = \frac{\varepsilon h^2 n^2}{\pi m Z e^2} \qquad \text{and} \qquad E_n = \frac{-Z^2 m e^4}{8h^2\varepsilon_0^2 n^2} = -\frac{Z^2 Rhc}{n^2}$$

If an electron makes a transition from n = n_2 to n = n_1

$$\frac{hc}{\lambda} = Z^2 Rhc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\Rightarrow \qquad v = \frac{Z}{n} v_0$$

$$r_0 = 0.53\,\mathring{A}; \qquad r = \frac{r_o n^2}{Z},$$

$$E_0 = -13.6\text{ eV}; \quad E = \frac{E_o Z^2}{n^2}$$

Also, K.E. = $-$P.E./2

E = P.E. + K.E. = P.E./2 = $-$K.E.

FINAL EXERCISE

1. In Rutherford's á-particle scattering experiment, what observation led him to predict the existance of nucleus?
2. Why did Rutherford assume that electrons revolve in circular orbits around the nucleus?
3. What is the ratio of the energies of the hydrogen atom in its first excited state to that its second excited state?
4. What is the SI unit of Rydberg's constant?
5. The Rydberg constant for hydrogen is 1096700 m^{-1}. Calculate the short and long wavelength limits of Lyman series.
6. How many times does the electron of H-atom go round the first orbit in 1s?
7. Describe Rutherford's scattering experiment and discuss its findings and limitations.
8. State the postulates of Bohr's model of atom.
9. Derive an expression of the energy of the electron in the nth orbit of hydrogen atom.
11. Calculate the radius of the third and fourth permitted orbits of electron in the hydrogen atom.
12. The energy transition in H-atom occurs from n = 3 to n = 2 energy level. Given $R = 1.097 \times 10^7\ m^{-1}$.

    (i) What is the wavelength of the emitted radiations?

    (ii) Will this radiation lie in the range of visible light?

    (iii) To which spectral series does this transition belong?
13. The ionisation potential of hydrogen is 13.6 volt. What is the energy of the atom in $n = 2$ state?
14. How are x-rays produced? Draw a labelled diagram to illustrate?
15. List the properties of x-rays and compare them with that of visible light.
16. How are continuous x-rays produced? What is the condition for the production of photons of highest frequency?

14 DUAL NATURE OF RADIATION & MATTER

Wave Particle Duality - Matter Waves and Particle Nature of Light

Despite the wave nature, electromagnetic radiations, have properties similar to those of particles. Thus, electromagnetic radiation emerges as an emission with a dual nature having both wave and particles aspects. In particular, the energy conveyed by an electromagnetic wave is always carried in units whose magnitude is proportional to frequency of the wave. These units of energy are called photons.

Energy of a photon is E = h.f, where h is Planck constant, and f is frequency of wave.

According to de-Broglie, nature loves symmetry; as wave behaves like matter, matter also behaves like wave. According to him, the wavelength of the matter wave is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$, where m is the mass and v is velocity of the particle.

If an electron is accelerated through a potential difference of V volt, then

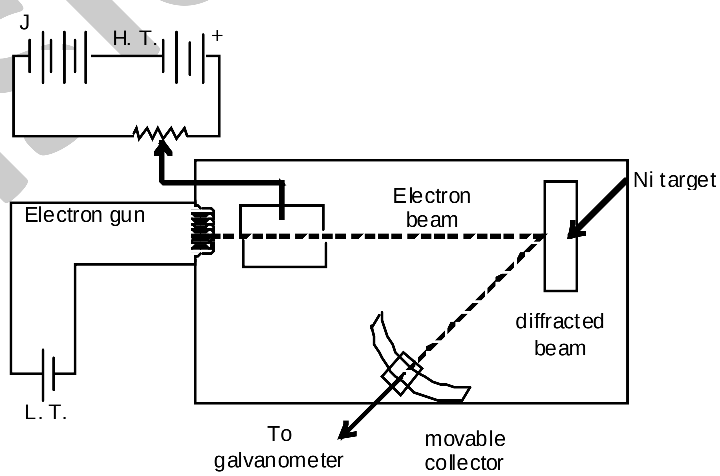
$$\frac{1}{2}m_0v^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m_e}} \quad \therefore \quad \lambda = \frac{h}{m_e v} = \frac{h}{\sqrt{2eVm_e}}$$

(It is assumed that the voltage V is not more than several tens of kV)

Davisson and Germer Experiment:

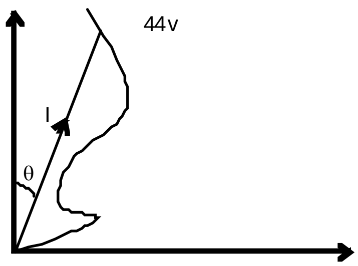
This experiment was conducted by Davisson and Germer in 1927. It shows the wave nature of electrons. In this experiment, an electron beam is accelerated through a high tension field and then bombarded on a Nickel crystal. The electrons are scattered due to diffraction (see figure). The scattered electrons are collected by a movable collector which is free to move along a graduated circular scale. Angle θ is measured by the scale.

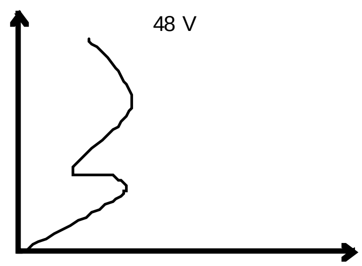
The intensity of diffracted electron beam is measured with the help of a galvanometer at different angles (θ).

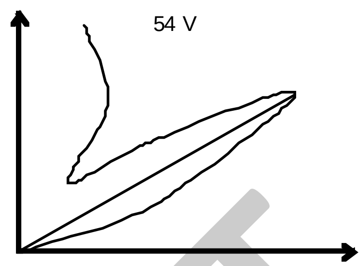


The intensity is plotted against θ for various values of accelerating voltage and the pattern observed is as shown below.

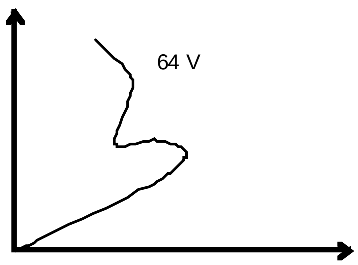
(A) (B) (C)

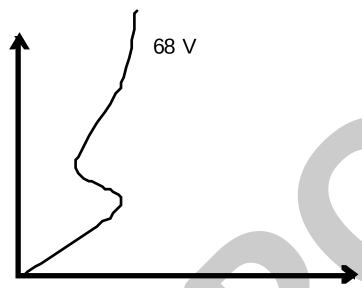






(D) (E)





It is observed that highest intensity is obtained at 54 V and $\theta = 50^0$. This diffraction maxima occurs due to constructive interference of electrons scattered by different layers of atoms in the metal plate. From electron diffraction measurements, the wavelength of the matter waves was found to be 0.165 nm. The de-Broglie's wavelength λ associated with electron is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mVe}} = \frac{1.227}{\sqrt{V}}\,\text{nm} = \frac{1.227}{\sqrt{54}} = 0.166\,\text{nm}$$

which is in excellent agreement with the results of the experiment.

Hence, the Davisson and Germer experiment confirms the wave nature of electrons and the de-Broglie relation.

Illustration 1. *What is the energy and wavelength of a thermal neutron at a temperature of 20^0 C.*

Solution: By definition, a thermal neutron is a free neutron in a neutron gas at about 20°C (293 K).

$$KE = \frac{3}{2}kT = \frac{3}{2}(1.38\times10^{-23})(293) = 6.07\times10^{-21}\,J$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0(KE)}} = \frac{6.63\times10^{-34}}{\sqrt{2\left(1.67\times10^{-27}\right)\left(6.07\times10^{-21}\right)}} = 0.147\,\text{nm}$$

Illustration 2. *An electron in a hydrogen like atom is in an excited state. It has a total energy of* J *3.4 eV. Calculate*

(i) the kinetic energy, and

(ii) the de-Broglie wavelength of the electron.

Solution: (i) U + K = E, where K = – E and U = 2E

$\therefore$ K = – (– 3.4 eV) = 3.4 eV

(ii) $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{6.63 \times 10^{-34}\ J-s}{\sqrt{2 \times \left(9.1 \times 10^{-31} kg\right) \times \left(3.4 \times 1.6 \times 10^{-19}\right)}} = 6.63Å$$

Photon Theory of Light

Light has wave character as well as particle character. Depending on the situation, one of the two characters dominates. The particles of light are called photons. Some of the important properties of a photon are given below.

(a) A photon always travels at a speed c = 3.0×10^8 m/s in vacuum and it is independent of any form of reference frame used to observe the photon.

(b) Rest mass of a photon is zero

(c) Each photon has a definite energy and a definite linear momentum.

$$E = hf = hc/\lambda \ \& \ p = h/\lambda = E/c$$

where h is Planck's constant & has a value 6.626×10^{-34} Js.

(d) A photon may collide with a material particle. The total energy and the total momentum remain conserved in such a collision. The photon may get absorbed and/or a new photon may be emitted. Thus number of photons may not be conserved.

(e) If the intensity of light of given wavelength is increased there is an increase in the number of photons crossing a given area in a given time. The energy of each photon remains the same.

Photoelectric Effect

When light of sufficiently small wavelength is incident on a metal surface, electrons are ejected from the metal. This phenomenon is called the photoelectric effect. The electrons ejected from the metal are called photoelectrons.

There are large number of free electrons in a metal which wander throughout the body of the metal. However these electrons are not free to leave the surface of the metal. As they try to come out of the metal, the metal attracts them back. A minimum energy, equal to the work function (W_0), must be given to an electron so as to bring it out of the metal. The electron after receiving the energy, may lose energy to the metal in course of collisions with the atoms of the metal. If electron is given an energy E which is greater than W_0 and it makes the most economical use of it, it will have a maximum kinetic energy.

$$KE_{max} = E - W_0.$$

Thus electrons with K.E. ranging from 0 to KE_{max} will be produced.

Laws of photoelectric effect

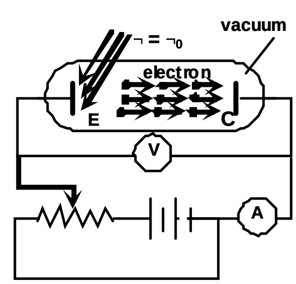
(i) The emission of photoelectrons is instantaneous.

(ii) The number of photoelectrons emitted per second is proportional to the intensity of the given incident light.

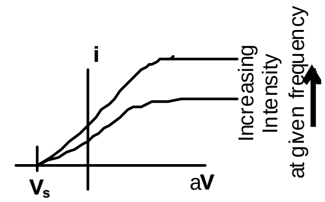
(iii) The maximum velocity with which electrons emerge is dependent solely on the frequency and not on the intensity of the incident light.

(iv) There is always a lower limit of frequency called threshold frequency below which no emission takes place, however high the intensity of the incident radiation may be.

The figure shown is an experimental arrangement for studying photoelectric effect. If we set up a suitable potential difference ΔV between E (Emitter) and surface C (collector), we can sweep the ejected photoelectrons and measure them as a photoelectric current I in the external circuit.



If the collector is at a higher potential than the emitter and if ΔV is large enough, the photoelectric current reaches constant saturation value at which all electrons emerging from emitter are collected. If we reduce ΔV to zero, the photoelectric current does not drop to zero because the electrons are emitted with definite range of speeds.



However, if we reverse the sign of the potential difference and make ΔV large enough, we eventually reach a point at which even the most energetic emitted electrons are turned back before they strike the collector and photoelectric current i does indeed drop to zero. The magnitude of this stopping potential difference is called the stopping potential V_S.

Conservation of energy gives,

$$eV_S = KE_{max}$$

Einstein's photoelectric equation

If the photon incident on a metal surface makes most efficient use of its energy and is just sufficient to liberate the electron. (with the Kinetic Energy of the electron being zero).

i.e. $hf_0 = W_0$

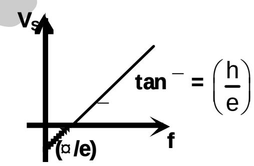
where f_0 is the threshold frequency and W_0 is the work function. If the frequency of incident light is less than f_0, no photoelectric emission takes place.

Now, suppose the frequency of incident photon is f.

Maximum Kinetic Energy of the photoelectron is

$\Delta KE_{max} = hf - hf_0 = h(f - f_0)$

$eV_s = hf$ ---- ϕ



$$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = \left[12400\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)eV\right].$$ where λ is in Angstroms (Å)

Illustration 3. *In a beam of light with wavelength 700 nm and intensity 100 W/m²*
(a) what is the momentum of each photon ?
(b) How many photons cross 2 cm² area perpendicular to the beam in one second ?

Solution: (a) Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{700 \times 10^{-9} \times 1.6 \times 10^{-19}} = 1.77 eV$$

$$p = \frac{E}{c} = \frac{1.77 eV}{3 \times 10^8 m/s}$$

$p = 5.9 \times 10^{-9}$ eV – s/m.

(b) Energy crossing 2 cm² in 1 sec.

$E' = 100 \text{ W/m}^2 \times 2 \times 10^{-4} \text{ m}^2 = 2 \times 10^{-2}$ J

No. of photons crossing 2 cm² in 1 sec.

$$= \frac{2 \times 10^{-2} J}{1.77 \times 1.6 \times 10^{-19} J/photon} = 7.06 \times 10^{16}$$

Illustration 4. *Photoelectric threshold of silver is ª = 3800Å. Ultra-violet light of ª = 2600Å is incident on silver surface. Calculate*
(a) the value of work function in joule and in eV.
(b) maximum kinetic energy of the emitted photoelectrons.
(c) the maximum velocity of the photo electrons.
(Mass of the electron = 9.11 ò 10^{-31} kg)

Solution: (a) λ_0 = 3800Å

$$W = hf_0 = h\frac{c}{\lambda_o} = \frac{6.63\times10^{-34}\times3\times10^{8}}{3800\times10^{-10}}J = 5.23 \times 10^{-19}J = 3.27\ eV$$

(b) Incident wavelength, λ = 2600Å

$$\therefore\ f = \text{incident frequency} = \frac{3\times10^{8}}{2600\times10^{-10}}Hz$$

Then $KE_{max} = hf - W_o$

$$hf = \frac{6.63\times10^{-34}\times3\times10^{8}}{2600\times10^{-10}} = 7.65\times10^{-19}J = 4.78\ eV$$

$KE_{max} = hf - W_0 = 4.78\ eV - 3.27\ eV = 1.51\ eV.$

(c) $KE_{max} = \frac{1}{2}mv_{max}^2$

$$\therefore\quad v_{max} = \sqrt{\frac{2\ \ KE_{max}}{m}} = \sqrt{\frac{2\times2.42\times10^{-19}}{9.11\times10^{-31}}} = 0.7289\times10^{6}ms^{-1}$$

SUMMARY

- **Wave–Particle Duality**

$$\therefore\quad \lambda_e = \frac{h}{m_e v} = \frac{h}{\sqrt{2eVm_e}}$$

- **Photoelectric Effect**

$$eV_s = K_{max} = \frac{hc}{\lambda} - \phi$$

where V_s = stopping potential, λ = wavelength of incident radiation, and ϕ = work function.

FINAL EXERCISE

1. In photoelectric emission, what happens to the incident photons?
2. What is the difference between a photon and a matter particle?
3. Why is the wave nature of matter not apparent in daily life?
4. How is velocity of photoelectrons affected if the wavelength of incident light is increased?
5. The threshold frequency of a metal is 5 × 10^{14} Hz. Can a photon of wavelength 6000Å emit an energetic photoelectron?
6. Does the threshold frequency for a metal depend on the incident radiations?
7. What are the various uses of photocell?
8. What was the aim of Davisson and Germer's experiment? On what principle does it depend?
9. Describe the experiment used for studying the photoelectric effect.
10. Explain the terms (a) Saturation voltage and (b)Stopping potential.
11. State the laws of photoelectric emission.
12. Describe the salient features of Einstein's theory of photoelectric effect.

15 NUCLEI & RADIOACTIVITY

The nucleus is positively charged and it is located at the centre of the atom. Almost the entire mass of the atom is concentrated in its nucleus.

Composition of Nucleus

A nucleus consists mainly of two types of particles, protons and neutrons. Collectively these two particles are sometimes referred to as nucleons.

(i) Atomic Number: It is the number of protons present in the nucleus. It is equal to the number of electrons in the atom.

(ii) Mass Number : Total number of protons and neutrons is the mass number.

If the atomic number of an atom is Z, and number of neutrons present in the nucleus of the atom is N, then its mass number can be given by the expression,

$A = Z + N$, where A = mass number.

Mass defect:

The actual mass of a nucleus is experimentally observed to be smaller than the sum of the masses of free nucleons constituting it. The difference between the experimental mass, $m({}_zX^A)$, of a nucleus and the sum of the masses of free nucleons (Z protons, and A-Z neutrons) is known as the mass defect :

$\Delta m = Zm_p + (A-Z)\, m_n - m\ ({}_ZX^A)$

Size of nucleus:

Experiments have shown that average radius R of a nucleus can be given by

$R = R_0\, A^{1/3}$

where $R_0 = 1.1 \times 10^{-15}$ m = 1.1 fm.

and A is the mass no. of the atom.

Volume of the nucleus can be calculated as below.

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$

It may be noted that density of the nucleus, $d = \frac{M}{A}$ is constant as $M \propto A$.

Illustration 1. *Calculate radius of nucleus ${}_{26}Fe^{56}$*

Solution: $R = R_0\, A^{1/x3} = 1.1 \times (56)^{1/3}$

$R = 1.1 \times 3.826$

$R = 4.208$ fm

Nuclear Forces: Nuclear forces operate between all the nucleons i.e. between proton - proton, neutron-proton and neutron – neutron.

- these forces are short ranged. They operate at distances of the order of femtometer or less.
- they are much stronger than electromagnetic forces (50 - 60 times)
- they are independent of charge.

Nuclear force depends not only on centre distance but also on spin. It is stronger if spins of nucleons are parallel.

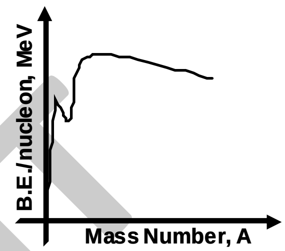
Binding energy

The amount of energy needed to disintegrate a nucleus into its constituent free nucleons is called the binding energy. It is the energy equivalent to the mass defect.

B.E. = Δmc^2, where c = velocity of light.

If mass is measured in amu then B.E. = ($\Delta m \times 931.5$)MeV. As 1 amu = 931.5 MeV/C^2

Binding Energy per nucleon



The binding energy per nucleon for a given nucleus is found by dividing its total binding energy by the number of nucleons it contains. The given figure show the binding energy per nucleon plotted against the number of nucleon for various atomic nuclei. The greater the binding energy per nucleon, the more stable the nucleus is. The graph has its maximum of 8.8 MeV/nucleon when number of nucleons is 56.

Two remarkable conclusions can be drawn from the curve in the figure shown. The first that if we can somehow split a heavy nucleus into two medium sized ones, each of the new nuclei will have more binding energy per nucleon than the original nucleus.

The other notable conclusion is that joining two light nuclei together to give a single nucleus of medium size also means more binding energy per nucleon in the new nucleus. Splitting a heavy nucleus is called Nuclear fission, while combining two nuclei is called Nuclear fusion.

Illustration2. *What is the binding energy of $_6C^{12}$?*

Solution: One atom of $_6^{12}C$ consists of 6 protons, 6 electrons and 6 neutrons. The mass of the uncombined protons and electrons is the same as that of six $_1^1H$ atoms (if we ignore the very small binding energy of each proton-electron pair).

Mass of six $_1^1H$ atoms = 6×1.0078	=	6.0468 u
Mass of six neutrons = 6×1.0087	=	6.0522 u
Total mass of component particles	=	12.0990 u
Mass of $_6^{12}C$ atom	=	12.0000 u
Mass defect = 12.0990 – 12.0000	=	0.0990 u
Binding energy = (931)(0.099)	=	92 MeV

Illustration 3. *Find out the binding energy per nucleon of an ~ particle in MeV. It is given that the masses of ~ particle, proton and neutron are respectively 4.00150 amu, 1.00728 amu and 1.00867 amu.*

Solution: An α particle consists of 2 protons and 2 neutrons.

Mass of constituents = $2 \times 1.00728 + 2 \times 1.00867$ = 4.0319 amu.

Mass defect = Δm = 4.0319 – 4.00150 = 0.0304 amu

Binding energy = 0.0304×931 MeV = 28.30 MeV

Radioactivity

Radioactivity was discovered in 1896 by Antoine Becquerel. Three extraordinary features of Radioactivity are:

1. When a nucleus undergoes α or β decay its atomic number Z changes and it becomes the nucleus of different element. Thus the elements are not immutable.
2. The energy liberated during radioactive decay comes from within individual nuclei without external excitation, unlike the case of atomic radiation.
3. Radioactive decay is a statistical process that obeys the laws of chance. No. cause-effect relationship is involved in the decay of a particular nucleus, only a certain probability per unit time.

Alpha Decay

An α - particle is a helium nucleus, i.e. a helium atom which has lost two electrons. It has a mass about four times that of a hydrogen atom and carries a charge +2e. They have very little penetrating power but have a very high ionising power. An α decay reduces the Z and the N of the original nucleus by two each.

Beta Decay

β-particles are electrons moving at high speeds. These have greater (compared to α- particles) penetrating power but less ionising power. Their velocities are close to the velocity of light. Unlike A particles, they have a wide spectrum of energies, i.e., beta particles possess energy from a certain minimum to a certain maximum value. In negative beta decay a neutron is transformed into a proton and an electron is emitted, this happens when nucleus has either too large a neutron/proton ratio for stability. In positive beta decay, a proton becomes a neutron and a positron is emitted.

Gamma rays (¥)

γ-rays are electromagnetic waves of wavelength of the order of ~ 10^{-11} m. They have the maximum penetrating power and the least ionising power. γ - rays are emitted due to the transition of an excited nucleus from a higher energy state to a lower energy state.

Soddy's displacement law

The emission of an ?particle reduces the atomic number by 2 and the mass number by 4 and, it displaces the element by two columns to the left in the periodic table.

$${}^{b}_{a}X = {}^{b-4}_{a-2}Y + \alpha$$

α – decay: ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

β – decay: ${}^{1}_{0}n \rightarrow {}^{1}_{1}P + {}^{0}_{-1}e^{-}$

β^{+} decay: ${}^{1}_{1}H \rightarrow {}^{1}_{0}n + {}^{0}_{1}e^{+}$

Electron capture: ${}^{1}_{1}H + {}^{0}_{-1}e^{-} \rightarrow {}^{1}_{0}n$

Radioactive decay law

In a radioactive decay the number of nuclei disintegrating per second $\left(\frac{dN}{dt}\right)$ is directly proportional to the number of undecayed nucleus atoms (N) present at that instant.

$\frac{dN}{dt} = -\lambda N$ where λ is the radioactivity decay constant.

If N_o is the number of radioactive nucleus present at a time t = 0, and N is the number at the end of time t, then $N = N_o e^{-\lambda t}$

The term $\left(-\frac{dN}{dt}\right)$ is called the activity of a radioactive substance and is denoted by 'A'.

Units of activity: 1 becquerel (Bq) = 1 disintegration per second (dps)

1 curie (Ci) = 3.7×10^{10} dps

1 rutherford = 10^{6} dps

Half life

The time interval during which a radioactive substance decays to half its original value. It is denoted by $T_{1/2}$.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Mean life

It is the average of the lives of all the nuclei. $T_{av} = \dfrac{\int_0^{\infty} N_0 e^{-\lambda t} dt}{N_0} = \dfrac{1}{\lambda} = \dfrac{T_{1/2}}{0.693}$

Illustration 4. *The half-life of Cobalt - 60 is 5.25 years. After what time would its activity have decreased to about one-eigth of its original value ?*

Solution: The activity is proportional to the number of undecayed atoms.
In each half-life, half the remaining sample decays.

Since, $\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{1}{8'}$, therefore, three half-lives or 15.75 years are required for the sample to decay to $\dfrac{1}{8}$th of its original strength.

Illustration 5. *The half-life of radium is 1620 years. How many radium atoms decay in 1s in a 1gm sample of radium. The atomic weight of radium is 226 kg/k mol.*

Solution: Number of atoms in 1gm sample is

$$N = \left(\frac{0.001}{226}\right)\left(6.02\times10^{26}\right) = 2.66\times10^{21} \text{ atoms.}$$

The decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(1620)\left(3.16\times10^{7}\right)} = 1.35\times10^{-11}\,s^{-1}$$

(taking one year $= 3.16 \times 10^7$ s)

$$\text{Now, } \frac{\Delta N}{\Delta t} = \lambda N = \left(1.35\times10^{-11}\right)\left(2.66\times10^{21}\right) = 3.6\times10^{10}\,s^{-1}$$

Thus, 3.6×10^{10} nuclei decay in one second.

Illustration 6. *A rock is 1.5 ò 10^9 year old the rock contains ^{238}U which disintegrates to from ^{206}Pb. Assume that there was no ^{206}Pb in the rock initially and it is only stable product formed by the decay calculate the ratio of numbers of nuclei of ^{238}U to that of ^{206}Pb in the rock. Half life of ^{238}U is 4.5 ò 10^9 years [$2^{1/3} = 1.259$]*

Solution: Let N_0 be the initial number of nuclei of ^{235}U.After time t

$$N_U = N_0\left(\frac{1}{2}\right)^n$$

Here n = number of half lives = $\dfrac{t}{t_{1/2}} = \dfrac{1.5\times10^9}{4.5\times10^9} = \dfrac{1}{3}$

$$\therefore N_U = N_0\left(\frac{1}{2}\right)^{\frac{1}{3}}$$

and $N_{pb} = N_0 - N_U = N_0\left[1-\left(\dfrac{1}{2}\right)^{1/3}\right]$

$$\frac{N_U}{N_{pb}} = \frac{(1/2)^{1/3}}{1-(1/2)^{1/3}} = 3.861$$

SUMMARY

- **Nuclear Physics**

(a) α decay $= {}^{Z}_{A}X \rightarrow {}^{Z-4}_{A-2}Y + {}^{4}_{2}He + \text{energy}$

(b) β^- decay $= {}^{Z}_{A}X \rightarrow {}^{Z}_{A+1}Y + \beta^- + \gamma + \text{energy}$

(c) β^+ decay $= {}^{Z}_{A}X \rightarrow {}^{Z}_{A-1}Y + \beta^+ + \gamma + \text{energy}$; where A = atomic number, Z = mass number

Energy Produced in a Nuclear Reaction

m_1 K_1 m_2 K_2 m_3 K_3 m_4 K_4

Initial **Final**

Initial energy: $E_i = m_1c^2 + m_2c^2 + K_1 + K_{-2} + hf_o$

Final energy: $E_f = m_3c^2 + m_4c^2 + K_3 + K_4 + hf$

Since $E_i = E_f$, therefore $Q = [(m_1 + m_2) - (m_3 + m_4)]c^2$

If Q is positive, the rest mass energy is converted to kinetic mass energy or radiation mass energy or both, and the reaction is exoergic.

If Q is negative, the reaction is endoergic. The minimum amount of energy that a bombarding particle must have in order to initiate an endoergic reaction, is called Threshold Energy E_{th}.

$E_{th} = -Q\left(\frac{m_1}{m_2} + 1\right)$, where m_1 = mass of the projectile and m_2 = mass of the target.

- **Binding Energy**

  It is the energy required to break up the nucleus into its constituent nucleus and place them infinitely apart at rest.

- **Radioactive Decay Law**

  If nuclei A is decaying to nuclei B

$$-\frac{dN}{dt} = \lambda N$$

$$\Rightarrow \qquad N = N_0e^{-\lambda t}$$

$$t = \frac{2.303}{\lambda}\log\frac{N_0}{N}$$

- **Activity:** $A = \left|\frac{dN}{dt}\right| = \lambda N$

- **Half Life**

  The time interval during which a radioactive substance decays to half its original value. It is denoted by $T_{1/2}$.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

- **Mean Life**

$$T = \frac{\int_0^{\infty} N_0e^{-\lambda t}dt}{N_0} = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693}$$

Units of activity: 1 becquerel (Bq) = 1 disintegration per second (dps)

1 curie (Ci) = 3.7×10^{10} dps

1 rutherford (rd) = 10^6 Bq = 10^6 dps

- **Chain Reaction**

Theoretically; it can be represented as

$$A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C$$

$$-\frac{dN_A}{dt} = \lambda_A N_A \qquad \text{. . . (1)}$$

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \qquad \text{. . . (2)}$$

$$\frac{dN_C}{dt} = \lambda_B N_B$$

Solving Eqs. (1), (2) and (3), we can get the values N_A, N_B, N_C at any time t.

FINAL EXERCISE

1. When does a radioactive sample disintegrate?

2. Differentiate between isotopes and isobars.

3. Explain the characteristics of binding energy per nucleon versus mass number curve.

4. What is the nature of nuclear force? Give its characteristics.

5. Explain how decay constant is related to half-life of a radioactive substance.

6. Define the following terms:

(i) Atomic number;

(ii) Mass number;

(iii) Mass defect;

(iv) Binding energy of nucleons;

(v) Half-life;

(vi) Average life;

(vii) Decay constant.

7. State the law of radioactive decay.

8. What is carbon dating? What is its importance?

16 NUCLEAR FISSION & FUSION

Nuclear reaction - Fission & Fusion:

A large amount of energy is liberated by either breaking a heavy nucleus into two nuclei of middle weight or by combining two light nuclei to form a middle weight nucleus. These processes are called nuclear fission and nuclear fusion respectively.

Nuclear Fission:

Middle weight nuclei are more tightly bound and have more binding energy. When a heavy nucleus breaks into two middle weight nuclei, binding energy increase. Hence the rest mass decreases. This extra energy is liberated as K.E. or some other form.

$$^{236}_{92}U \rightarrow \, ^{137}_{53}I + ^{97}_{39}y + 2n$$

and $$^{238}_{92}U \rightarrow \, ^{140}_{56}Ba + ^{94}_{39}Kr + 2n$$

Nuclear Fusion

Light weight nuclei are less tightly bound than middle weight nuclei. When two light weight nuclei combine and form a middle weight nucleus, binding energy is increased and rest mass is decreased. This energy is liberated as K.E. or some other form.

Conservation law's in Nuclear Reactions.

1. Conservation of Mass Energy: Mass energy of the reactants is equal to mass energy of the products.
2. Conservation of Linear Momentum: The sum of momentum of the reactants and products remains conserved.
3. Conservation of mass number: Total mass number does not change in nuclear reactions.
4. Conservation of electric charge: Total electric charge remains conserved.

Energy of a nuclear reaction

Initial energy $E_i = m_1c^2 + m_2c^2 + K_1 + K_{-2} + hf_o$

Final energy $E_f = m_3c^2 + m_4c^2 + K_3 + K_4 + hf$

Since $E_i = E_f$

$\therefore \quad [(m_1 + m_2) - (m_3 + m_4)]c^2 = (K_3 + K_4) - (K_1 + K_2) + hf - hf_o$

The energy that is released or absorbed in a nuclear reaction is called the Q - value or disintegration energy of the reaction.

$$Q = [(m_1 + m_2) - (m_3 + m_4)]c^2$$

If Q is positive, rest mass energy is converted to kinetic mass energy, radiation mass - energy or both, and the reaction is exogenic.

If Q is negative, the reaction is endoergic. The minimum amount of energy that a bombarding particle must have in order to initiate an endoergic reaction, is called threshold Energy E_{th}

$$E_{th} = -Q\left(\frac{m_1}{m_2} + 1\right)$$

Where m_1 = mass of the projectile and m_2 = mass of the target

Illustration 1. *Find the Kinetic energy of the ~. particle emitted in the alpha decay of $_{88}Ra^{226}$.*
[Given : m (^{228}Ra) = 226.0254024 u, m (^{222}Rn) = 222.017571 u, m (^{4}He) = 4.00260 u]

Solution: From the question we have

$$ {}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + \alpha $$

$$ Q = [m\,(^{226}Ra) - m(^{222}Rn) - m(^{4}He)]c^2 $$
$$ = [\,226.0254024 - 222.017571 - 4.00260\,] \times (931.5\ MeV)/u $$
$$ = 4.871\ MeV $$

Illustration 2. *Neon - 23 decays in the following way :*
$_{10}Ne^{23}$ ì $_{11}Na^{23} + {}_{-1}e^{0} + \bar{\nu}$
Find the minimum and maximum KE that the particle can have. The atomic masses of ^{23}Ne and ^{23}Na are 22.9945 u and 22.9898 u, respectively.

Solution: Since the mass of an atom includes the masses of the atomic electrons, the appropriate number of electron masses must be subtracted from the given values.

Mass of Reactant	**Mass of Products**
${}^{23}_{10}Ne$ 22.9945 – 10m_e Total mass = 22.9945 – 10 m_e	22.9898 – 11m_e, ${}^{0}_{-1}e$ – m_e Total mass = 22.9898 – 10 m_e

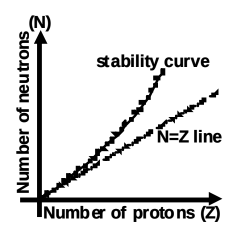
Mass defect = 22.9945 - 22.9898 = 0.0047 u

Q = (0.0047)(931) = 4.4 MeV

The β - particle and anti - neutrino share this energy. Hence the energy of the β-particle can range from 0 to 4.4 MeV.

Stable Nuclei:

A stable nucleus maintains its constitution all the time. The figure shows a plot of neutron number (N) versus proton number (Z) for the nuclides observed. For light stable nuclides, the neutron number is equal to the proton number so that ratio N/Z is equal to 1. The ratio N/Z increases for the heavier nuclides and becomes about 1.6 for heaviest stable nuclides. Protons are positively charged and repel one another electrically. This repulsion becomes so great in nuclei with more than 10 protons or so that an excess of neutrons, which produce only attraction forces, is required for stability. Thus stability curve departs more and more from N = Z line as Z increases.



FINAL EXERCISE

1. On the basis of B.E per nucleon versus mass number curve, explain nuclear fusion.
2. What is a nuclear reaction? State the conservation laws obeyed in nuclear reactions. Give threes examples of nuclear reactions.
3. What is nuclear fission? Give an example to illustrate your answer.
4. Calculate the mass of $_{235}U$ consumed to generate 100 mega watts of power for 30 days.
5. What is nuclear fusion? Write an equation of nuclear fusion to support your answer.
6. What is the source of energy in the sun? How is it generated? Illustrate with an example.

17 SEMICONDUCTORS & SEMICONDUCTING DEVICES

Introduction

There is a large number of free electrons in a conductor which wander randomly in the whole of the body. Whereas in an insulator all the electrons are tightly bound to same nucleus or the other. Free electrons experience force due to an electric field $\vec{E}$ established inside a conductor and acquire a drift speed. This result in an electric current. The ratio of resulting current density $\vec{J}$ and the electric field $\vec{E}$ is the conductivity σ. Larger the conductivity σ, better is the material as a conductor. The relation between these quantities is $\vec{J} = \sigma\vec{E}$.

$$\rho = \frac{1}{\sigma}$$

Further on the basis of the relative values of electrical conductivity (σ) or resistivity ($\rho = 1/\sigma$) the solids are broadly classified as:

(a) **Metals:** They posses high conductivity or very low resistivity

$\sigma \sim 10^2 - 10^8\ sm^{-1}$

$\rho \sim 10^{-2} – 10^{-8}\ \Omega m$

(b) **Insulators:** They have low conductivity and high resistivity.

$\sigma \sim 10^{-8}\ sm^{-1}$

$\rho \sim 10^8\ \Omega$ m.

(c) **Semicoudnctors:** They have conductivity or resistivity intermediate to metals and insulators.

$\sigma \sim 10^{-5} - 10^0\ sm^{-1}$

$p \sim 10^5 – 10^0\ \Omega m$

In this chapter, we will restrict ourselves to the study of inorganic semiconductors, particularly elemental semiconductors Si and Ge.

There is another classification scheme on the basis of the source and nature of the charge carriers. Such a scheme divides semiconductors as intrinsic and extrinsic semiconductors.

(a) Intrinsic Semi-Conductors: These are pure semi conductor materials (impurity less than 1 part in 10^{10}). The electrical conduction is by means of mobile electrons and holes (hole as positive charge carriers)

(b) Extrinsic Semi-Conductors: These are obtained by adding or doping the pure semiconductor material with small amounts of certain specific impurities with valency different from that of the atoms of the parent material. Consequently, the number of mobile electrons / holes gets drastically changed. So, the electrical conductivity in such materials is essentially due to the foreign atoms or in other words extrinsic in nature.

Electrical Conduction in Semiconductors

The energy of an electron in an isolated atom is decided by the orbit in which it is revolving but because of the presence of many atoms placed close to each other these electron energies would be different.

In a solid, the atoms are held together closely in a well defined 3-dimensional array or lattice by strong forces as the separation between them is quite small. At such separations, the outer electronic orbits of the neighbouring atoms overlap considerably and hence get significantly distorted. In fact, some valence electrons may be shared by several atoms and it becomes difficult to say which electron goes with which atom. In conducting metals like Al, Cu etc. the outer orbit electrons are shared by all the atoms and the metallic crystal can be visualised as positive inner cores embedded in a regular fashion in a sea of shared electrons as shown in fig. 1

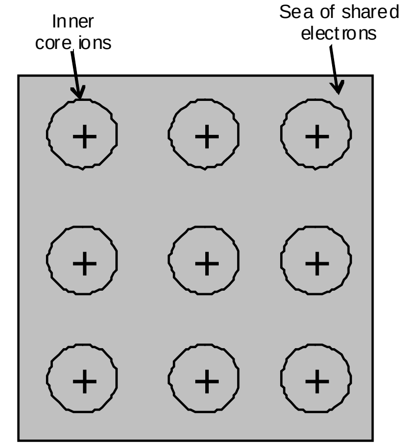


Fig. 1
(Inner core ions are embedded at fixed positions in the sea of shared electrons)

Such electron are obviously free to travel (free electrons) throughout the material randomly. The number of mobile electrons will be of the order of ~ 10^{29} /m^3 which is very large. A small electric field results in the flow of these free electrons in the direction of the +ve potential and gives low resistivity (or high conductivity to metals)

In semiconductors at low temperatures the conductivities of most of the semiconductors is low and comparable to those of insulators but at higher temperature give moderate values of conductivity.

Valance - Bond Description: Semiconductors Ge and Si whose lattice structure is shown in figure -2. Each atom is surrounded by four nearest neighbours as shown in the dotted curve. Si and Ge have four valence electrons and share one of its four valence electrons with each of its four nearest neighbour atoms. These shared electron pairs are referred to as forming a covalent bond or simply a valence bond.

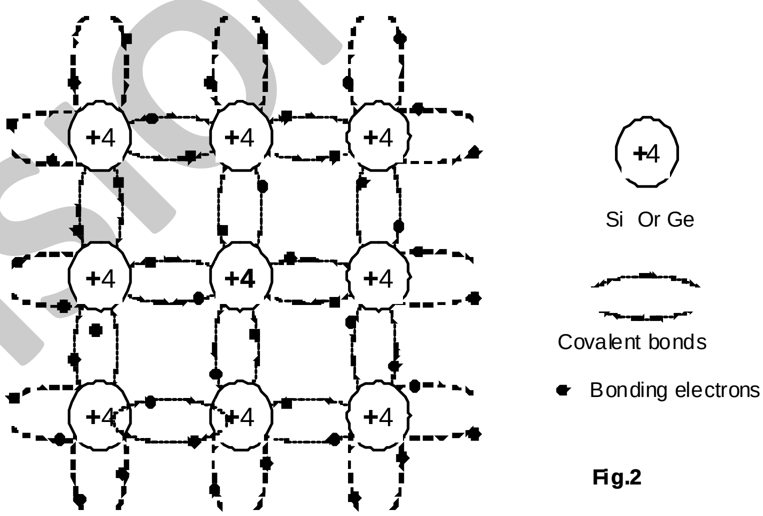


Fig.2

As the temperature increases, some of these electrons may break –away (becoming free electrons and leaves a vacancy with the effective positive electronic charge is called a hole.)

In intrinsic semiconductor, the number of free electrons (n_e) is equal to the number of holes (n_h). In semiconductors apart from electrons, the holes also move. The motion of hole may be looked upon as a transfer of ionisation from one atom to another carried out by the motion of the bound electrons between their covalent bonds . The total current is the sum of the electron current I_e due to the thermally generated conduction electrons and the hole current I_h.

$$I = I_e + I_h$$

Valance bond description of Extrinsic semiconductors

In extrinsic semiconductors a small amount say, ~ 1 ppm of suitable impurity is added to the pure semiconductor. The deliberated addition of a desirable impurity is called doping and the impurity atoms are called dopants. A size of the dopant and the semiconductor atmost should be nearly same so that dopant does not distort the original pure semiconductor lattice and preferably substitute some original semiconductor atom.

There are two types of dopants used in doping semiconductors like Si or Ge.

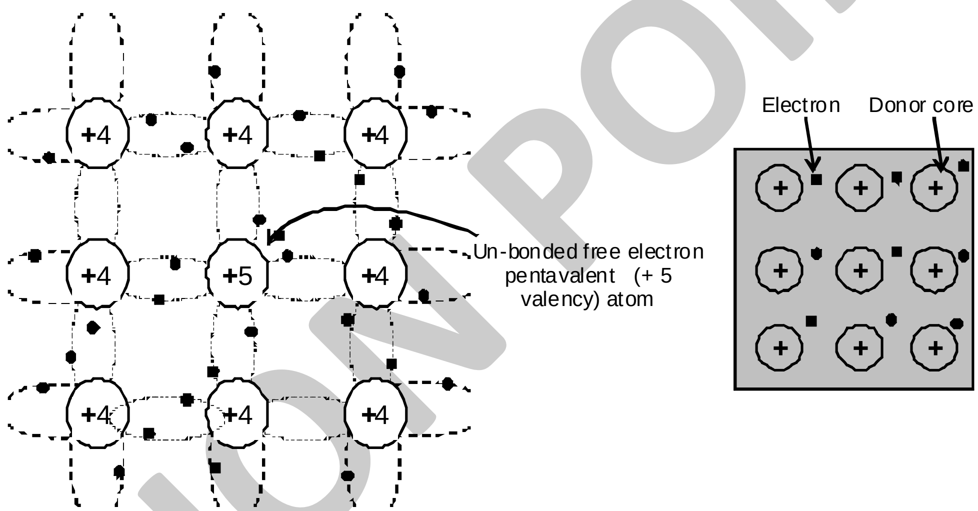
(i) Trivalent (Valency 3) like. Indium (In), Boron(B), Aluminium (Aℓ) etc.

(ii) Pentavalent: (Valency 5): like Arsenic (As), Antimony (Sb), Phosphorus (P) etc.

The pentavalent and trivalent dopants in Si or Ge give two entirely different types of semiconductors.

(a) n–type semiconductor:

If we dope Si or Ge (valancy 4) with a pentavalent (valency 5) element as shown in figure -3.



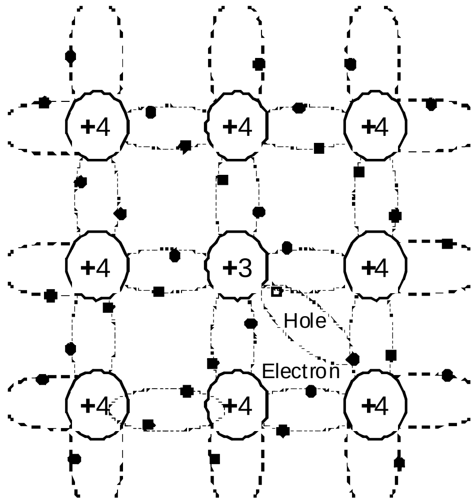
N-type semiconductor with one additional effective positive charge and its associated extra electron

Fig. 3
Schematic representation of n-type semiconductor

Four of its electrons make bond with the four silicon neighbours while the fifth electron is free to move and only a very small ionisation energy is required to set free this extra electron. Obviously the numbers of conduction electrons are now more than the number of holes. Hence the majority carriers are negatively charged electrons. These materials are known as n-type semiconductors.

(b) p- type semiconductor:

When Si or Ge (tetravalent) is doped with group - III trivalent impurities like Al, B, In etc. as shown in figure - 4. The dopant has one outer electron less than Si or Ge. This atom fails to form bond on one side. Therefore some of the outer bound electrons in the neighbourhood have a tendency to slide into this vacant bond leaving a vacancy or hole at its own site. Therefore these holes are in addition to the thermally generated holes. Thus, for such a material the holes are the majority carriers.



p-type semiconductor with one effective additional negative charge and its associated holes.

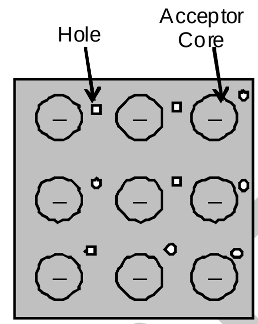


Fig. 4
Schematic representation of p-type semiconductor

Note : that the crystal maintains an overall charge neutrality.

Note:

(i) In a doped semiconductor, the number density of electrons and holes is not equal. But it can be established that

$$n_e\, n_h = n_i^2$$

where n_e and n_h are the number density of electrons and holes respectively and n_i is number density of intrinsic carriers (i.e. electrons or holes) in a pure semiconductor.

(ii) In n-type semiconductor, the number density of electrons is nearly equal to the number density of donar atoms N_d and is very large as compared to number density of holes. Hence

$$n_e \approx N_d >> n_h$$

(iii) In p-type semiconductor, the number density of holes is nearly equal to the number density of acceptor atoms N_a and is very large as compared to number density of electrons. Hence

$$n_h \approx N_a >> n_e$$

Illustration 1. *Pure Si at 300 K has equal electron (n_e) and hole (n_h) concentration of 1.5 ò 10^{16} m^{-3}. Doping by indium increases n_h to 4.5 ò $10^{22} m^{-3}$. Calculate n_e in the doped silicon.*

Solution: In a doped semiconductor, the number density of electrons and holes is not equal. But it can b established that

$$n_e n_h = n_i^2$$

n_e = number density of electrons

n_h = number density of holes

n_i = number density of intrinsic carries (i.e. electron or holes in a pure semiconductor)

$$n_e = \frac{n_i^2}{n_h} = \frac{\left(1.5\times10^{16}\right)^2}{\left(4.5\times10^{22}\right)} = 5\times10^9\, m^{-3}$$

Energy band description of solids - Metals, Insulators and Semiconductors:

Due to strong overlapping of the orbitals. It is difficult to ascribe any one electron (or hole) belonging to any particular atom. Electron wave motion is in fact, quantum mechanical in nature is more closely related with the energy and momentum concepts rather than the space - localisation and velocity concepts so far used in the bond picture. This alternative approach is termed as energy - band description of solids.

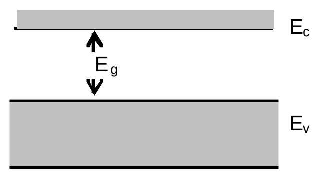


Fig. 5

The lower band is called the valence band which is completely filled. The upper band is called the conduction band which is completely empty.

At equilibrium spacing between the lowest conduction band energy is E_c and highest valence band energy is E_v. Above E_c or below E_v there are a large number of closely spaced energy levels as show in figure 6. Pauli's exclusion principle says that maximum number of two electrons can be in each energy level.

This gap between the top of the valence band and bottom of the conduction band is called the energy band gap. It may be large small or zero depending upon the material.

In case of conductors, E_g > 3eV, insulators, because electrons cannot be easily excited from the valence band to the conduction band by any external stimuli.

In case of $E_g = 0$, where the conduction and valence bands, are overlapping. This situation makes a large number of electrons available for electrical conduction and the resistance of such materials is low or the conductivity is high.

E_g < 3 eV is the case of finite but smaller band gap exists. Because of the small band gap, some electrons can be thermally excited to the conduction band and can move in the conduction band. This is the case of semiconductors. All the three cases are shown in figure -6.

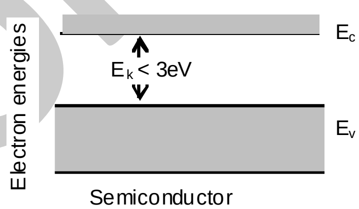
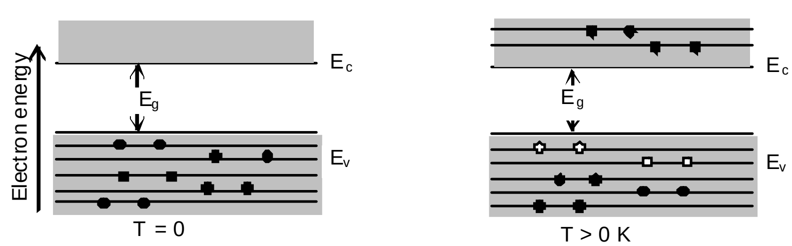
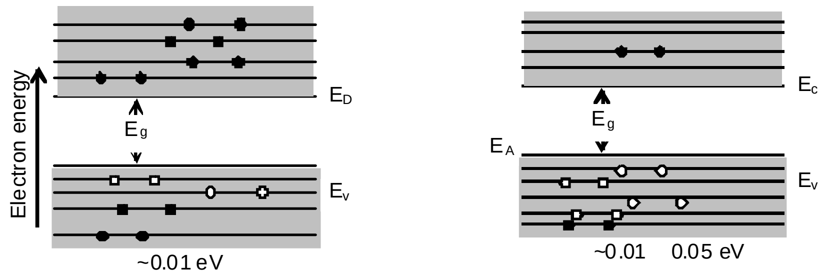


Fig. 6

An intrinsic semiconductor will behave like an insulator at T = 0 K. At higher temperature thermally excited electrons, partially occupy some states in the conduction band. Therefore, the energy band diagram of an intrinsic semiconductor will be as shown in figure - 7.



(a)



(b) (c)

Fig. 7

In the case of extrinsic semiconductor, additional energy states, apart from E_C and E_v due to donor impurities (E_D) and acceptor impurities (E_A) also exist. We have already discussed that in the n – type semiconductor very small energy (~ 0.1 eV) is required for the electrons to be released from the donor impurity.

Similarly, for p - type semiconductor an acceptor energy level E_A is obtained as shown in figure. The position of E_A is very near to the top of the valence band because an electron added to the acceptor impurity to complete its bonding within the semiconductor structure, comes easily from the valence band electrons of some other semiconductor atoms in the lattice. The above description is grossly approximate and hypothetical but it helps in understanding the difference between metals, insulators and semiconductors.

PN Junction

A p-n junction is basic building block of almost all semiconductor devices, a single p-n junction acts as a rectifying diode, two such junctions (viz. p – n followed by n – p) makes p-n-p transitor. Therefore detailed understanding of the p-n junction is required before going into the details of various other devices.

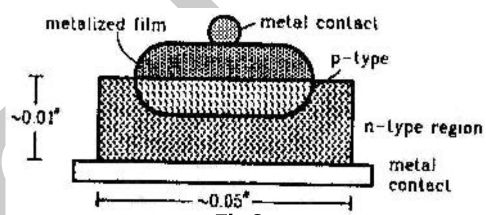


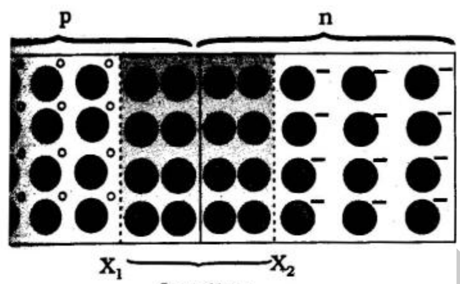
Fig.8

Diffusion current: Because of the concentration difference, holes try to diffuse from p-side to the n-side. However, the electric field at the junction exerts a force on the holes towards the left as they come to the depletion layer. Only these holes with a high kinetic energy are able to cross the junction. Similarly those electrons which start towards the left with high kinetic energy are able to cross the junction. This diffusion results in an electric current from the p-side to the n-side known as diffusion current.

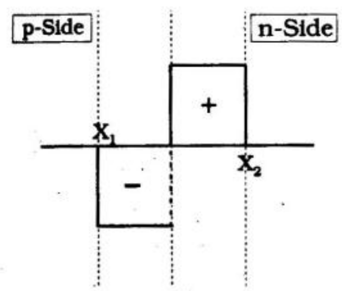
Drift Current: Occasionally a covalent bond is broken (because of thermal collisions) and the electron jumps to the conduction band. An electron-hole pair is thus created. Also occasionally a conduction electron fills up a vacant bond so that an electron – hole pair is destroyed. These processes continue in every part of the material. However, if an electron hole pair is created in the deplection region, there is almost no chance of recombination of a hole with an electron in the deplection region because the electron is quickly pushed by the electric field towards the n-side and the hole towards the p-side. As electron – hole pairs are continuously created in the depletion region, there is a regular flow of electrons towards the n-side and of holes towards the p-side. This makes a current from the n-side to the p-side. This current is called the drift current.

Description of p- n junction without external applied voltage of Bias:

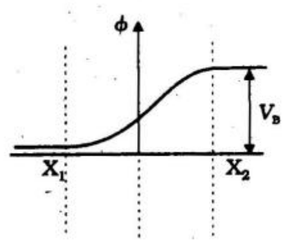
Suppose a p- n junction has just been formed. There are more electrons on the n –side while the number of holes on the p-side is larger. Because of concentration gradient holes from the p-side will go towards the n-side and electrons from n – side will diffuse towards the p-side. Diffusion of holes towards the right and diffusion of electrons towards the left make the right half positively charged and the left half negatively charged. This creates an electric field near the junction from the right to the left and will opposes any further diffusion of the majority carriers from either sides. This potential acts as a barriers and opposes any further diffusion of the majority carriers from either sides. Hence is known as Barrier Potential (V_B)



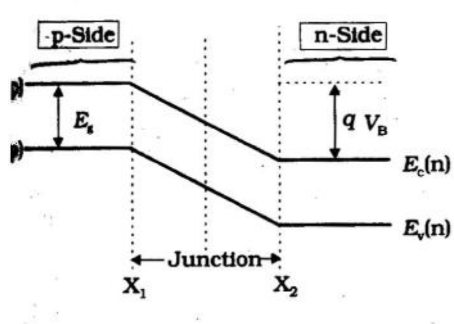
(a)



(b)



(c)

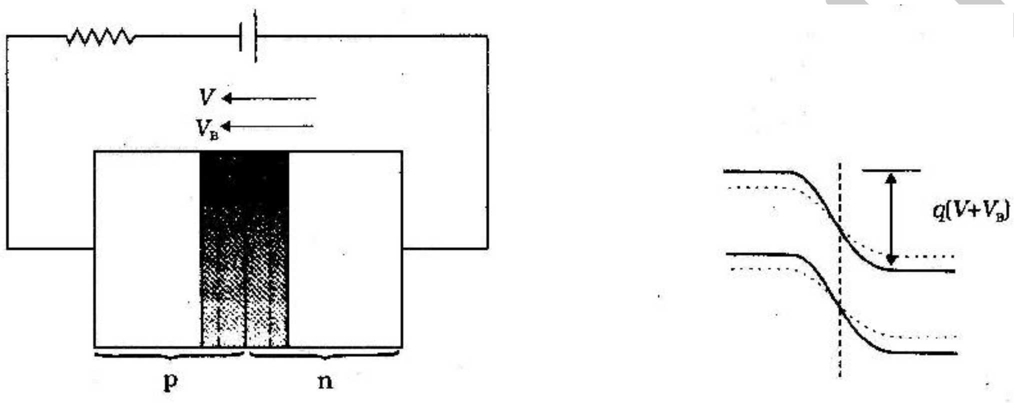


(d)

Behaviour of p – n junction with an external applied voltage or Bias:

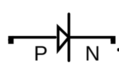
If the positive terminals of the battery is connected to the p-side and the negative terminal to the n-side, junction is forward biased. Due to the forward bias connection, the potential of the p-side is raised and hence the height of the potential barrier decreases. The width of the depletion region is also reduced in forward bias. Diffusion current increases by connecting a battery in forward bias. The drift current remains almost unchanged because the rate of formation of new electron hole pairs is fairly independent of the electric field unless the field is too large.

Reverse bias: The applied voltage V on the n-side is positive and is negative on the p-side, the junction is said to be reverse biased. In this case, the potential barrier becomes higher as the battery further raised the potential of the n-side. The width of the depletion region is increased Diffusion becomes more difficult and hence the diffusion current decreases. The drift current is not appreciably affected and hence it exceeds the diffusion current.



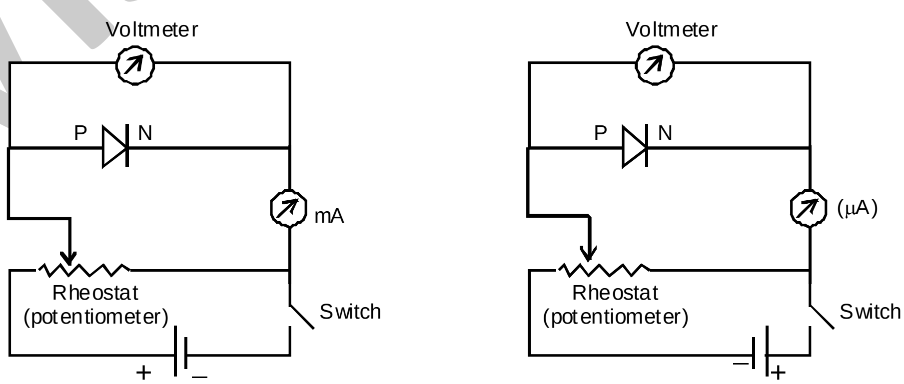
(a) (b)

Voltage- current (V – I) characteristics of a p-n junction diode

In the p-n junction discussed above, there are two electrode connections one on the p-side and another on the n-side. Hence it is generally called diode (di + ode; di – means two and ode come from electrode). The diode is symbolised as 

. The arrow points from the p-side to the n-side. This junction offers a little resistance if we try to pass an electric current from the p-side to the n-side and offers a large resistacne if the current is passed from the n-side to the p-side.

The circuit arrangements for studying the V-I characteristic of a diode is shown in figure. The voltage applied to the diode can be changed through a potentiometer.



and for different values of voltages, the value of the current is noted.

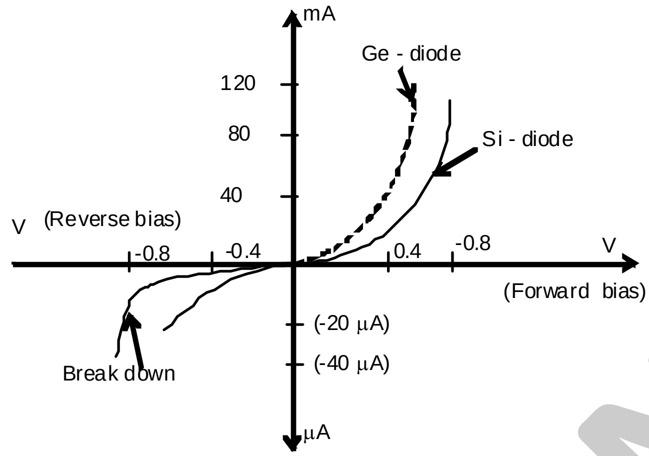


Figure shows a qualitative plot of current versus potential difference for a p - n junction. This is known as an I -V characteristic of the p- n junction. Note that the scales are different for the positive and negative current. In forward biasing, the current first increases very slowly almost negligible till the voltage across the diode crosses a certain value. After this diode current increases significantly. This voltage is called the threshold voltage or cut in voltage (~ 0.2 V for germanium diode and ~ 0.7 V for silicon diode)

For the diode in reverse bias the current is very small (~ μA) and almost remains constant with bias. It is called reverse saturation current. However for special cases, at very high reverse bias (break down voltage) the current suddenly increases. The general purpose diodes are not used beyond the reverse saturation current region. Primarily p – n diode restricts the flow of current only in one direction (forward bias)

Dynamic Resistance or A.C. Resistance of the Junction Diode:

It is defined as the ratio of a small change in voltage Δv applied across the junction to a small change in junction current ΔI .i.e.

$$R_d = \frac{\Delta v}{\Delta I}$$

Photonic p-n junction Devices

We can have semiconductor electronic devices, in which the light photons have also a role to play in the overall performance of the device. Such devices are called photonic or opto – electronic devices. Further they can be classified as:

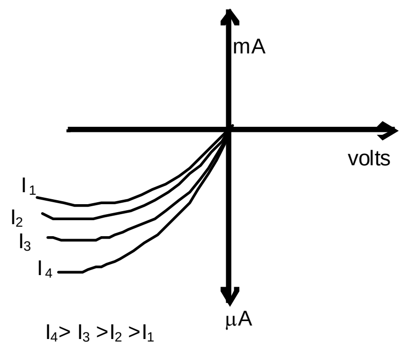
(i) Photo -detectors for detecting optical signals (e.g. photodiodes and photo-conducting cell)

(ii) Photo voltaic devices for converting optical radiation into electricity (e.g. solar cells)

(iii) Devices for converting electrical energy into light (e.g., light emitting diodes and diode lasers)

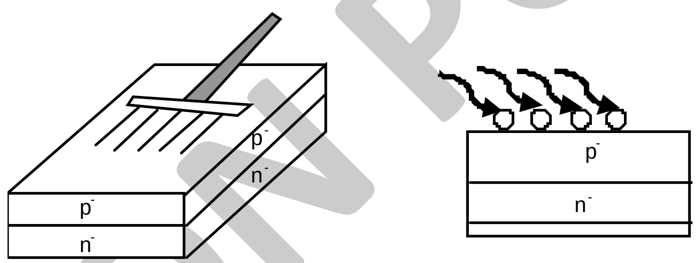
(a) Photodiodes: The general principle of all semiconductor based photo-detectors is the electron excitation from the valence band to the conduction band by photons. Suppose an optical photon of frequency ν. Corresponding energy $h\nu$ is incident on a semiconductor and its energy is greater than the band gap of the semiconductor (i.e $h\nu > E_g$). This photon will excite an electron from the valence band to the conduction band. Thus an electron hole pair is generated. These are additional charge carriers termed as photogenerated charge carriers which obviously increase the conductivity of the semiconductor. Increase in conductivity of the semiconductors is proportional to incident intensity of light.. Therefore, by measuring the change in the conductance of the semiconductor, one can measure the intensity of the optical signal. Such photoelectrons are known as photoconductive cells. However, more commonly used photodetecting devices are photodiodes.

The diodes are generally reverse biased when used as photodiode. Suppose a p-n diode is illuminated with .... $\nu > E_g$ and intensities I_1, I_2, I_3, etc. There would be a change in the reverse saturation current as shown in figure. Hence, a measurement of the change in the reverse saturation current on illumination can give the values of the light intensity.



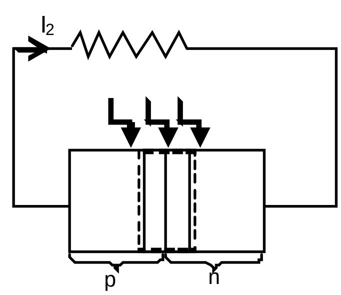
(b) Solar cell: Solar cell is based on similar principle as junction photodiode (generation of voltage due to the bombardment of optical photons).

Let us take a simple p-n junction solar cell shown in figure. A n-type semiconductor substrate (backed with a current collecting metal electrode) is taken, over which a thin p-layer is grown (e.g. by diffusion of a suitable acceptor impurity or by vapour deposition).



On top of the p-layer, metal finger electrodes are prepared so that there is enough space between the fingers for the light to reach p-layer (and the underlying p-n junction) to be able to produce photo-generated holes and electrons.

When light (with $h\nu > E_g$) falls at the junction, electron-hole pairs are generated which move in opposite directions due to the junction field. They would be collected at the two sides of the junction giving rise to a photo-voltage.



When external load is connected as shown in figure, a photo-current I_2 flows. The materials most commonly used for solar cells are silicon (Si) and gallium arsenide (Ga As).

(c) Light emitting Diode (LED)

When a conduction electron makes a transition to the valence band to fill up a hole in a p-n junction, the extra energy may be emitted as a photon. If the wavelengths of this photon is in the visible range(380 nm – 780 nm), one can see the emitted light. Such a p-n junction is known as light emitting diode abbreviated as LED.

The semiconductor used in LED is chosen according to the required wavelength of emitted radiation. Visible LED's are available for red, green and orange.

LED's have the following advantages over conventional incandescen lamps.

(i) Low operational voltage and less power

(ii) Fast action and no warm up time required.

(iii) Long life and ruggedness

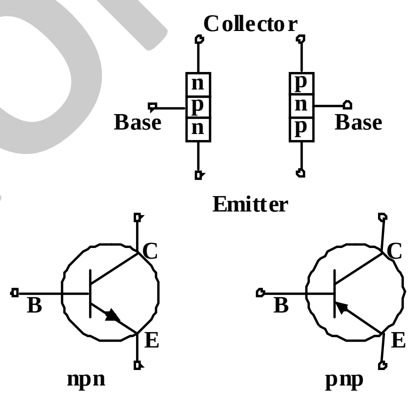
Transistor

It is a three layer semiconductor device consisting of either two N- and one P-type semiconductors or two P-type and one N- type. In former case it is called N-P-N transistor (P-type semiconductor is sand wiched between two N-type semiconductors). In the latter case it is called P-N-P transistor (N-type semiconductor is sandwiched between two P-type semiconductors). Three distinct regions thus formed are called emitter, base and collector. Transfer of a current from a low resistance to a high resistance circuit produces the basic amplifying action of transistor. A combination of the words " transfer and resistance" gave the term "transistor".

Transfer + resistor = Transistor

In actual design, the middle layer is very thin ($\approx 1\ \mu$m) as compared to the widths of the two layers at the sides. The middle layer is called the base and is very lightly doped with impurity. One of the outer layers is heavily doped and is called emitter. This supplies a large number of majority carriers for current flow through the transistor. The other outer layer is moderately doped and is called collector. Which collects a major portion of the majority carriers supplied by the emitter. Emitter is of moderate size and collector is larger in size as compared to the emitter.

Terminals come out from the emitter, the base and the collector for external connections. Thus, a transistor is a three- terminal device. Figure shows the symbols used for a junction transistor.



Basic transistor circuit configurations and transistor characteristics

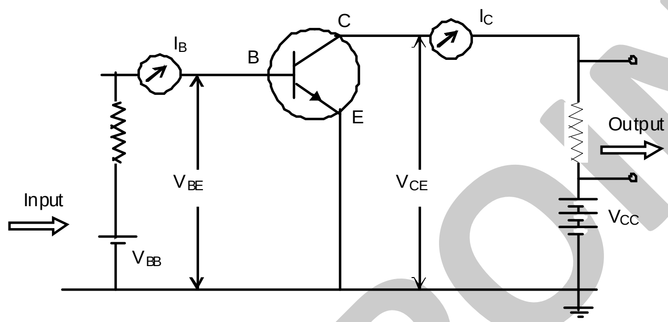
In any electronic circuit or device there has to be two terminals for input and two terminals for out put, In a transistor, only three terminals are available viz. Emitter (E), Base (B) and collector (C). Therefore in a circuit the input/output connections have to be such that one of these (E, B or C) is common to both the input and the output. Therefore, the transistor can be connected in either of the three following configurations.

(a) Common Emitter (CE) (b) Common base (CB) (c) Common collector (CC)

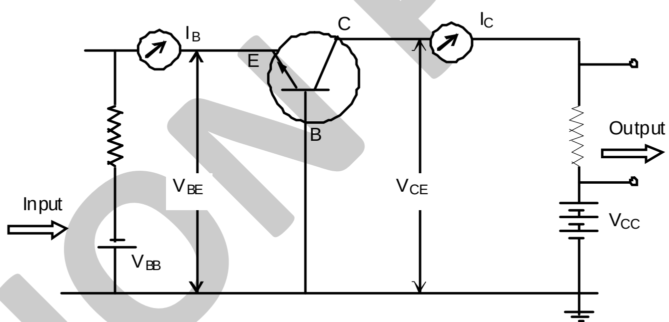
In normal operation of a transistor, the emitter – base junction is always forward – biased, whereas the collector - base junction is reverse –biased. The arrow on the emitter line shows the direction of the current through the emitter –base junction. In an n-p-n transistor, there are a large number of conduction electrons in the emitter and a large number of holes in the base. If the junction is forward –biased, the electrons will diffuse from the emitter to the base and holes will diffuse from the base to the emitter. The direction of electric current at this junction is therefore, from the base to the emitter. This is indicated by the outward arrow on the emitter line in figure. Similarly, for a p-n-p transistor the current is from the emitter to the base when this junction is forward - biased which is indicated by the inward arrow in figure.

Biasing: To operate the transistor suitable potential differences should be applied across the two junctions is called biasing of the transistor.

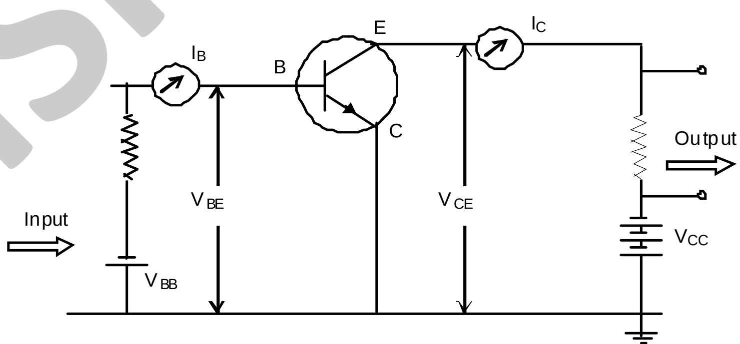
Common Emitter, Common Base and Common Collector configuration are shown in figure, (a), (b) and (c) respectively.



(a)



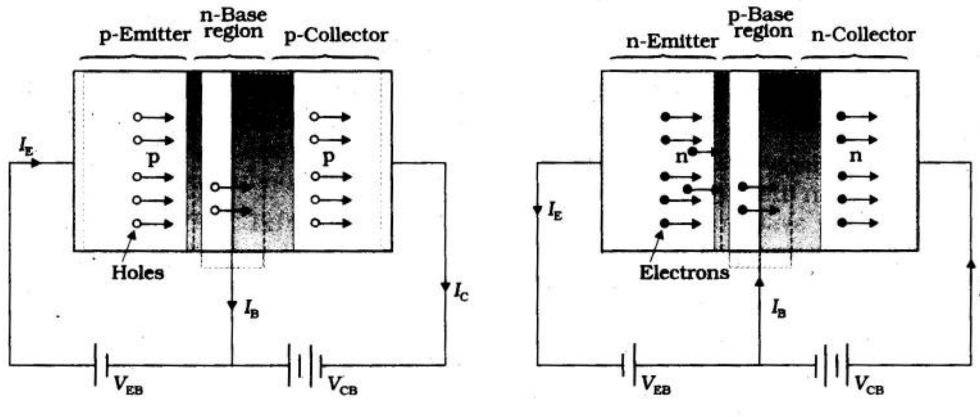
(b)



©

In common – emitter mode, the emitter is kept at zero potential and the other two terminals are given appropriate potentials. Similarly in common- base mode, the base is kept at zero potential, whereas in common – collector mode, the collector is kept at zero potential but we shall restrict ourselves only to CE configuration.

Consider the case of a biased p-n-p transistor shown in figure



(a) (b)

Bias Voltage applied : (a) p-n-p transistor and (b) n-p-n transistor

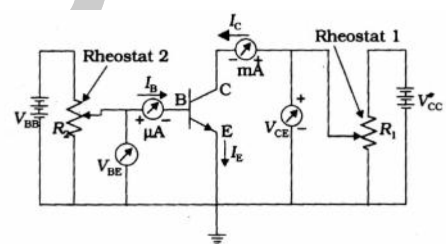
As the emitter base junction is forward biased, a large number of holes (majority carriers) from p-type emitter block flow towards the base. These holes have a tendency to combine with the electrons in the n- region of the base. Only a few holes (less than 5 %) are able to combine with the electrons in the base-region (giving only a very small base current , I_B) because the base is lightly doped and very thin. In the collector region, these holes see the favourable negative potential at the collector and hence they easily reach the collector terminal to constitute the collector current I_C. From Kirchhoff's current law.

$$I_E = I_C + I_B \qquad (I_C >> I_b)$$

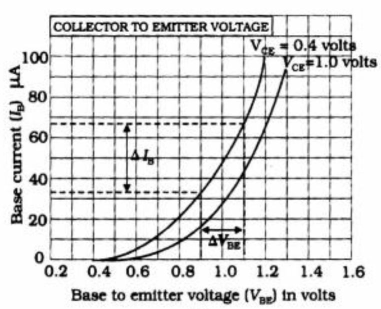
Similar description can be made for a biased n-p-n transistor as shown in figure. Here, the electrons are the majority carriers supplied by the n-type emitter region.

Any variation in the voltages on the input and output sides results in a change in the input and output currents. The variation of current on the input side with input voltage (I_E versus V_{BE}) is known as input characteristics while the variation in the output current with output voltage (I_C versus V_{CE}) is known as output characteristics.

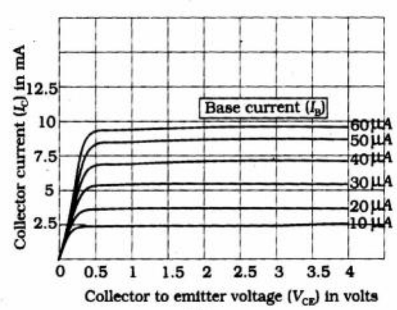
A simple circuit for drawing the input and output characteristics of an n-p-n transistor is shown in figure.



(a)



(i)



(ii)

The voltage applied to the base – emitter junction i.e. in the input section is V_{BE}. When the input current I_B is plotted against the voltage V_{BE} between the base and the emitter. The input characteristics shown in figure are like those of a forward – biased p-n junction. If the biasing voltage is small as compared to the height of the potential barrier at the junction, the current I_B is very small. Once the voltage is more than the barrier height, the current rapidly increases. However, since most of the electrons diffused across the junction go to the collector, the net base current is very small even at large values of V_{BE}.

To draw the output characteristics, we change the value of V_{CE} and note the values of I_C. For small values of the collector voltage, the collector – base junction is reverse biased because the base is at a more positive potential. The current I_C is then small. As the electrons one forced from the emitter side, the current I_C is still quite large as compared to a single reverse biased p-n junction. As the voltage V_C is increased, the current rapidly increases and becomes roughly constant once the junction is forward – biased. For higher base currents, the collector current is also high and increases more rapidly even in forward bias.

Input output characteristics are used to calculate the important transistor parameters as follows.

(i) Input Resistance (r_i) This is defined as the ratio for change in base-emitter voltage (ΔV_{BE}) to the resulting change in base current (ΔI_B) at constant collector – emitter voltage (V_{CE}).

$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$$

The value of r_i is of the order of a few hundred ohms.

(ii) Output resistance (r_0): This is defined as the ratio of change in collector –emitter voltage (ΔV_{CE}) to the change in collector current (ΔI_C) at constant base current I_B.

$$r_0 = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B=\text{constant}}$$

The values of r_0 are very high (of the order of 50 to 100 kΩ)

(iii) Current amplification factor ( ): This is defined as the ratio of the change in collector current (output current) to the change in base current.

$$\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CB}}$$

This is also known as current gain. We normally work in the region in which I_C is almost independent of V_{CE} (or varies very slowly with V_{CE}). This is called the active region.

SUMMARY

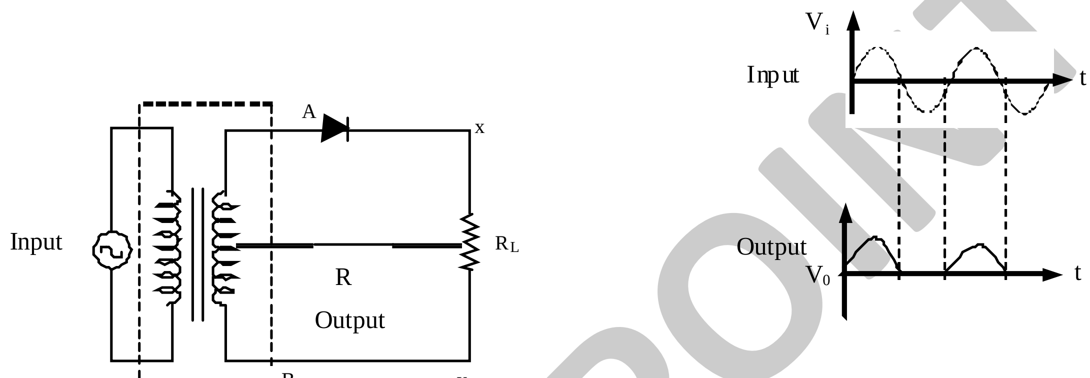
- Semiconductors are materials like silicon (Si) and germanium (Ge), which have conductivities midway between insulators and conductors.
- Semiconductors are of two types : Intrinsic (pure) and extrinsic (dopped).
- Extrinsic semiconductors can be *p*-type (dopped with 3rd group impurities) or *n*-type (doped with 5th group impurities).
- A *p-n* junction diode consists of a *n*-type region and a *p*-type region, with terminals on each end.
- When a *p-n* junction is formed, diffusion of holes and electrons across the junction results in a depletion region which has no mobile charges.

- The ions in the region adjacent to the depletion region generate a potential difference across the junction.
- A forward biased *p-n* junction offers low resistance to flow of electrons.
- A reverse biased *p-n* junction diode offers high resistance to flow of current.
- A *p-n* junction allows current to flow in only one direction.
- There are various types of diode e.g. photo diode light emitting diode, Zener diode and solar cell.
- A photo diode is always connected in reverse bias.
- A transistor consists of three separate regions (emitter, base and collector) and two junctions. Emitter is most heavily doped and base is the least doped.
- While collector has the largest size, base is the thinnest.
- Transistor can either be *n-p-n* type or *p-n-p* type.
- A transistor can be connected in any of the three configurations: common collector (*CE*), common base (*CB*) or common emitter (*CE*).
- The characteristics of a transistor vary according to the configuration of the transistor.
- *CE* configuration is preferred over other configurations as it provides high current gain and voltage gain.

18 APPLICATION OF SEMICONDUCTOR DEVICES

p- n diode as a rectifier:

The forward bias resistance is low as compared to the reverse bias resistance this diode property has been used to restricts the voltage variation of ac to one direction only, a phenomenon known as rectification. A simple rectifier circuit called half wave rectifier, using only one diode is shown in figure



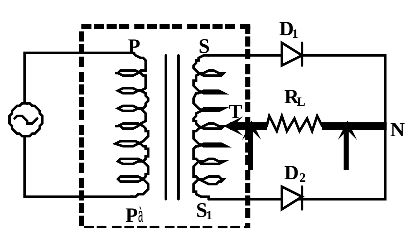
Half Wave Rectifier

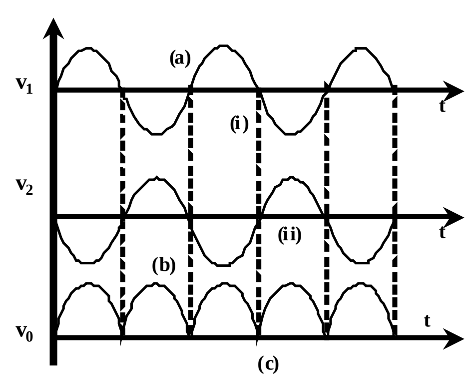
The secondary of the transformer supplies the desired ac voltage across A and B. The diode is forward biased when the voltage at A is positive and it conducts. Diode is reversed biased, when A is negative and it does not conduct. Therefore, in the positive half cycle of ac there is a current through the load resistor R_L and we get an output voltage as shown in figure.

The output voltage, though still varying, is restricted to only one direction and is said to be rectified. We get a voltage in the output for only one half cycle. Therefore such a circuit is known as Half Wave Rectifier.

Using two diodes as shown in the circuit arrangement gives rectified voltage corresponding to the positive as well as negative half of the ac cycle. Therefore it is known as Full Wave Rectifier. The circuit uses centre tap transformer and two diodes. The secondary of the transformer is would in to two equal parts as shown in the figure.

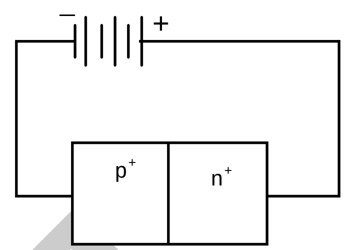
Note that voltage at any instant at 'A' (input of diode D_1) and B (input of diode D_2) with respect to the centre tap are out of phase with each other. This means when the diode D_1 gets forward biased and conducts (while D_2 is not conducting). Hence during positive half cycle we get an output current (and a consequent output voltage across the load resistor R_L) as shown in figure. At another instant, when the voltage at A becomes negative then the voltage at B would be +ve.



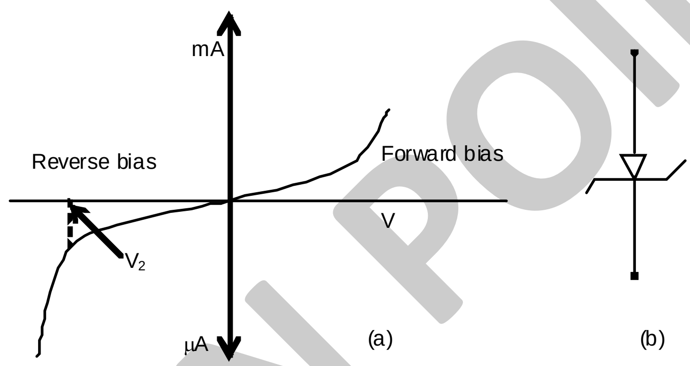


Now diode D_2 conducts and the diode D_1 does not conduct giving an output current and output voltage (across R_L) during the negative half cycle of the input ac. Thus, we get output voltage during the +ve as well as the –ve half of the cycle (during the full wave). This circuit is known as FULL WAVE RECTIFIER. Obviously this is more efficient circuit for getting rectified voltage or current.

Zener Diode: If the reverse –bias voltage across a p-n junction diode is increased. The holes in the n-side and the conduction electrons in the p-side are accelerated due to the reverse-bias voltage. If these minority carrriers acquire sufficient kinetic energy from the electric field and collide with a valence electron the bond will be broken and the valence electron will be taken to the conduction band. Thus a hole electron pair will be created. Breakdown occurring in this manner is called avalanche breakdown.



Breakdown may also be produced by direct breaking of a valance bonds due to high electric field. When breakdown occurs in this manner it is called zener breakdown. At this voltage the rate creation of hole – electron pairs is increased leading to the increased current.

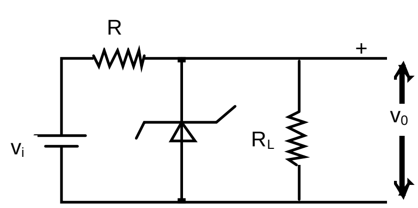


(a) V – I characteristics of a Zener Diode.

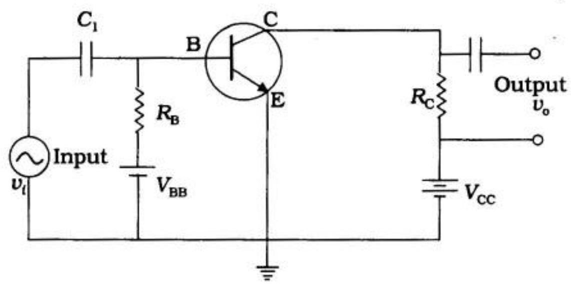
(b) The symbolic representation of a Zener diode

Once the breakdown occurs, the potential difference occurs the diode does not increases even if the applied battery potential is increased. This characteristic of diode is used to obtain constant voltage output. In other words, for widely different Zener currents, the voltage across the Zener diode remains constant.

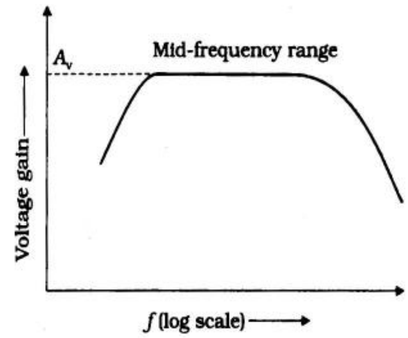
Figure shows a typical circuit which gives constant voltage V_0 across the load resistance R_L. Even if there is a small change in the input voltage V_i, the current through R_L remains almost the same. The voltage across zener diode remais essentially the same for the change in current through the diode.



Transistor as an Amplifier (CE – configuration) : Figure shows an amplifier circuit using an n-p-n transistor in common – emitter mode. The battery v_{BB} provides the biasing voltage V_{BE} for the base – emitter junction. A potential difference V_{CC} is maintained between the collector and the emitter by the battery V_{CC}. The input signal voltage V_i is connected to the input side through a capacitance so that the biasing dc voltage V_{BB} is blocked from going towards the source of signal. Similarly the dc voltage V_{CC} in the output is blocked with the help of capacitor C_2.



(a)



Without signal, a dc current I_B flows through R_B white I_C is the dc collector current. If V_i is applied to the input base – emitter side, it will change I_B to $I_B + i_B$ where i_B is due to the signal voltage V_i. The collector current would also change to $I_C + i_C$ where i_C is the collector current due to the input signal. The effective input signal (due to v_i or i_B) to the transistor is the voltage across R_B (input resistance)

$$V_i = i_B R_B$$

The output signal voltage (v_0) across R_c would be $v_0 = i_C R_C$.

Therefore, the voltage gain (A_v) of the amplifier (output signal divided by input signal voltage) is given by

$$A_v = \left(\frac{v_0}{v_i}\right) = \left(\frac{i_c R_c}{i_B R_B}\right)$$

i_c and i_B are respective collector and base current due to the signal input voltage. Further we know $\beta = \frac{i_C}{i_B}$

or, $A_v = \beta(R_C/R_B)$

Thus, the voltage gain is related to the current amplification factor of the transistor, externally connected collector and base resistances. The above expression is without considering the effect of transistor parameters like base – emitter resistance, base collector resistance, junction capacitances etc.

Power gain = voltage gain × current gain

$$= \beta^2 \frac{R_C}{R_B}$$

Illustration 1. *An n-p-n transistor in a common emitter mode is used as a simple voltage amplifier with a collector current of 4 mA. The terminal of an 8 V battery is connected to the collector through a resistance R_B. The collector emitter voltage V_{ce} = 4V, base emitter voltage V_{be} = 0.6 V and base current amplification factor $_{dc}$ = 100. Calculate the values of R_L and R_B.*

Solution: Potential difference across $R_L = 8V - V_{ce} = 8 - 4 = 4V$

Now $I_c R_L = 4V$

or, $R_L = \frac{4}{4\times10^{-3}} = 10^3\,\Omega = 1k\Omega$

Here, $I_b = I_c/\beta = \frac{4\times10^{-3}}{100} = 4 \times 10^{-5}$ A

Potential difference across R_B is $= I_b R_B = 8 - V_{be} = 8 - 0.6 = 7.4$ V or, $R_B = \dfrac{7.4}{I_b} = \dfrac{7.4}{4 \times 10^{-5}} = 1.85 \times 10^5\ \Omega = 185\ k\Omega.$

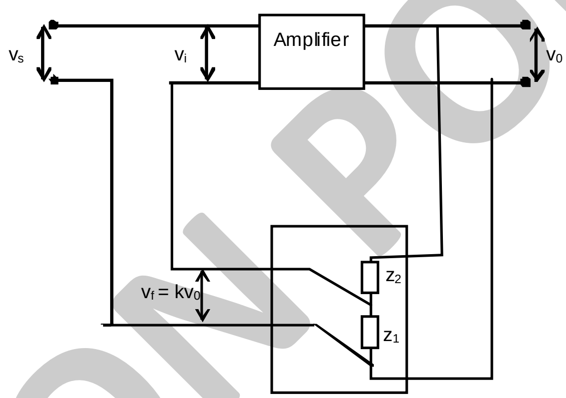
Transfer conductance: Transfer conductance g_m is defined as

$$g_m = \frac{\Delta I_C}{\Delta V_{BE}}$$

Which indicates how output current will change with change in V_{BE}. Further to have a large amplification, a small change in V_{BE} should result in a large change in the collector current I_C.

Transistor as an oscillator: In an amplifier, we have seen that input signal (ac) appears as an amplified output signal. and an external input was necessary for an amplifier. In an oscillator, we get ac output without any external input signal for same a portion of the output voltage of an amplifier is feedback to its input terminal in phase, this process is termed as positive feedback as shown in figure. The voltage across the z_1 is feedback in the input of oscillator.

We consider the circuit in which the feedback is accomplished by inductive coupling from one coil winding (T_1) to another coil winding (T_2). Coils T_2 and T_1 are wound on the same core therefore inductively coupled through their mutual inductance. For simplicity reason detailed biasing circuits have been emitted.



The feedback can be achieved by inductive coupling
(through mutual inductance or LC or RC networks)

To understand how oscillations are built. Suppose switch S_1 is put on to apply proper bias for the first time. Current flows through the coil T_2 and increases from X to Y as shown in figure. Current flows in the emitter circuit because of inductive coupling between coil T_2 and coil T_1 (positive feed back) This current in T_1 (emitter current) also increases from X′ to Y′. The current in T_2 (collector circuit acquires the value y. When the transistor becomes saturated and there is no further change in collector current. Now there will be no further feedback (field becomes static) from T_2 to T_1. The emitter current begins to fall and decreases from Y downwards z. A decrease of collector current causes the magnetic field to decay around the coil T_2. Thus T_1 is now seeing a decaying field in T_2 (opposite from what it saw at the initial start operation). Thus causes a further decrease in the emitter current fill it reaches z when the transistor is cut off. Now both I_E and I_C cease to flow and the transistor has reverted back to its original state. The whole process now repeats itself. That is the transistor is driven to saturation to cut – off and back is determined by the constants of the tank circuit or tuned circuit i.e. inductance L of coil T_2 and C connected in parallel to it. The resonance frequency (f) of this tuned circuit determines the frequency at which the oscillator will oscillate

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Illustration 2. *The current gain for common emitter amplifier is 59. If the emitter current is 6.0 mA, find the base current and collector current.*

Solution: Here, $\beta = 59$; $I_c = 6.0$ mA;

$I_b = ?$ and $I_c = ?$

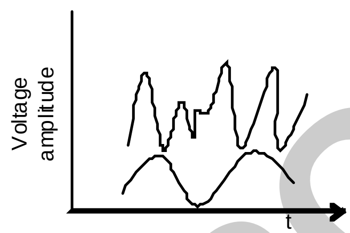
$$\beta = \frac{I_c}{I_b} = \frac{I_e - I_b}{I_b}$$

or, $$I_b = \frac{I_c}{1+\beta} = \frac{6.0}{1+59} = 0.1\,\text{mA}$$

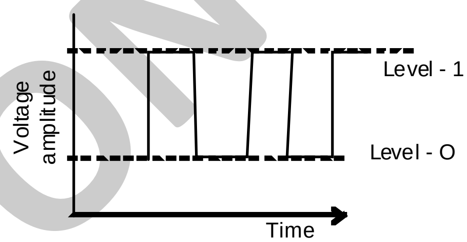
$I_c = I_e - I_b = 6.0 - 0.1 = 5.9$ mA.

Digital Electronics and logic Gates:

The wave form shown in figure, a continuous range of values of voltages are possible.



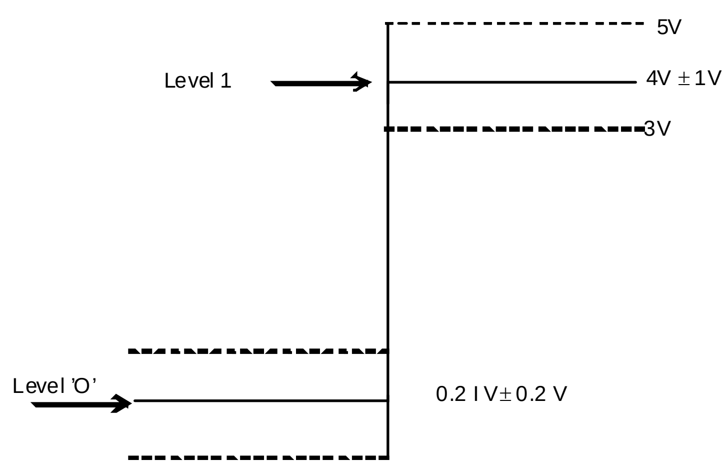
These are analog signals but in a pulse waveform in which only discrete values of voltages are possible. The high level is termed as '1' while the low level is called '0'. Further we know there are a number of questions which have only two answers either yes or No. There are a number of objects which can remains in either of two states only. An electric bulb can either be ON or OFF.



Above is closely related to the binary system of digits which we are already familiar.

Using the two levels of a signal like that in figure to represent binary digits 0 and 1 (called bits).

Please remember that 0 and 1 levels are not the same as a 0 and 1 V. In practice the bit 0 or 1 is recognised by the presence or absence of the pulse(i.e. either at high or at low levels). Further, the digital high and low levels are specified in certain voltage levels as shown in figure.



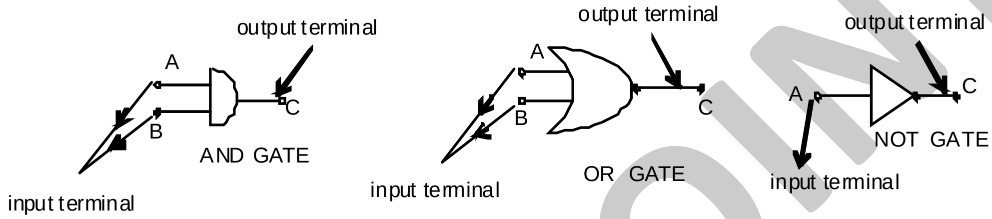
Logic Gates:

A logic gate evaluates a particular logical function using an electronic circuit. The circuit has one or more input terminals and an output terminal. A fixed positive potential V (say + 5V) denotes the logical value 1 and a potential zero (in general equal to the earth's potential) denotes the logical value 0. If zero potential is applied to an input terminal, the corresponding independent variable takes the value 0'.

If the positive potential V is applied to the terminal, the corresponding variable takes the value 1. The output terminal denotes the value of the function. Value of the function is O if the potential is zero. It is 1 if the potential is V.

A gate may have more than one output terminals then each output terminal represents a separate function and the same circuit may be used to evaluate more than one functions.

Figure shows the symbols for different logic gates with the terminals shown on the left are the input terminals and the terminals on the right is the output terminals.



AND Gate:

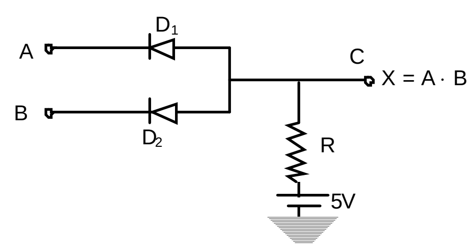
An AND gate has two or more inputs and one output. The output C of AND gate is 1 if all the inputs is simultaneously have the state 1. We say that inputs A and B are ANDed to get C. It is also denoted by the symbol of dot. Thus C = A. B. C is a function of A and B, If we give the value of C for all possible combinations of A and B, the function is completely specified.

Truth table for AND function is as given below.

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

Realisation of AND gate with diodes:

Figure shows the construction of AND gate with two p-n junction diodes.



Suppose the potential at A and B are both zero so that both the diodes are forward biased. The potential at C is equal to the potential at A or B because forward biased diode offer no resistance. Thus C = 0. Now suppose, A = 0 and B = 1, Now diode D_1 is forward biased. the potential at C is equal to the potential at A which is zero. Thus, if A = 0 and B = 1 then C = 0. Similarly, when A = 1 and B = 0 then C = 0.

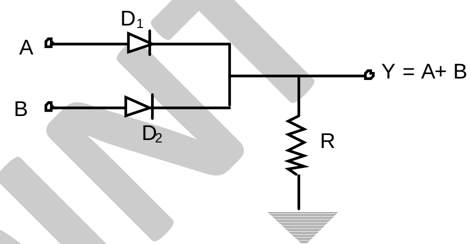
Now suppose A = B = 1. Now both diodes are reversed biased. As the diodes are not conducting, there will be no current through R and the potential at X will be equal to 5 V,

i.e. C = 1

OR Gate:

An OR gate has two or more inputs with one output. The output C will be 1 when the input A or B or both are 1. It is also represented by the symbol of plus. Thus C = A + B. This statement can also be given in the form of a table known as truth table as given below.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



Realisation of OR gate with diodes: Figure shows the construction of an OR gate using two p-n junction diodes.

If input A = B = 0, there is no potential difference any where in the circuit so Y = 0. If A = 1 and B = 0. The diode D_1 is forward –biased and offers no resistance. Therefore potential at C is equal to the potential at A, i.e., 5V. Same is the case if A = 0 and B = 1, then Y = 1.

It both A and B are 1, both the diodes are forward - biased and the potential at Y is the same as the common potential at A and B which is 5 V. This also gives Y = 1.

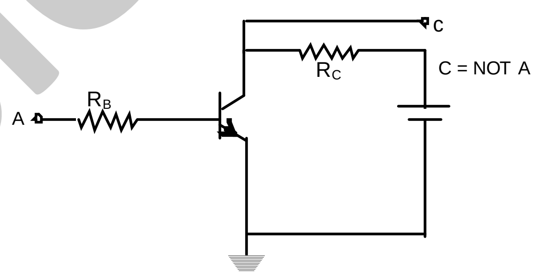
NOT Gate: It has a single input (A) and a *single output* (C). The output is not the same as the input. It performs a negation operation on the input (output is the inverse of the input). The truth table is given below.

Truth table of NOT gate

A	C
0	1
1	0

Realisation of NOT gate:

A NOT gate can not be constructed with diodes. For realising this, we use a simple transistor inverter circuit given in figure.



If A = 0, the emitter – base junction is unbiased and there is no current through it.

Therefore there is no current through the resistance R_C and the potential at C is 5 V.

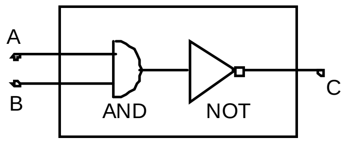
Thus, if A = 0, C = 1. Please note that the collectors base junction is also reverse – biased.

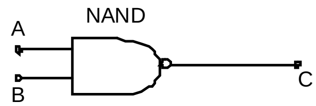
If the potential at A is 5V, the base – emitter junction is forward biased and there is a large current in the circuit. There is a potential drop across R and its value at C becomes zero.

Thus if A = 1, C = 0.

NAND gate:

It is a combination of NOT and AND gates in which the negation operation (NOT) is applied after AND gate as shown in figure. This simply means that for input $C = \overline{A \cdot B}$

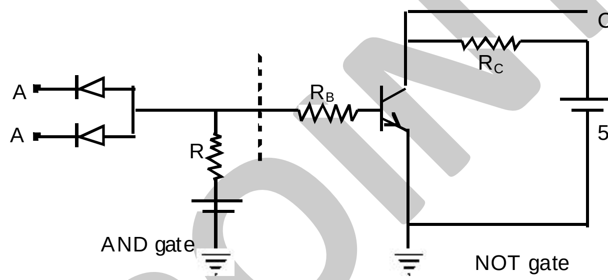




Conditions giving 1 output in AND gate will give O output in NAND gate and vice-versa. Hence truth table for NAND gate will be as under.

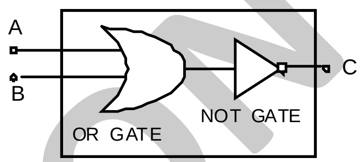
Truth table of NAND gate

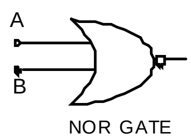
A	B	C
0	0	1
0	1	1
1	0	1
1	1	0



Circuit diagram of NAND gate

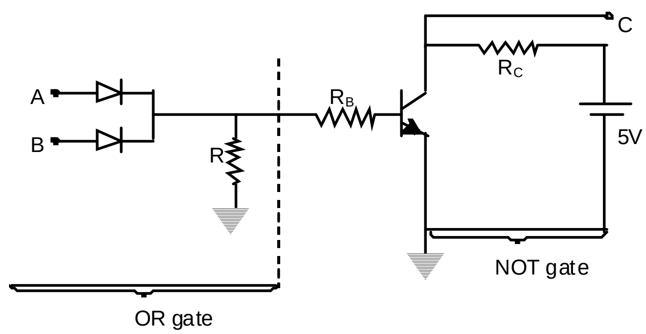
NOR Gate: A negation (NOT-operation) applied after OR gate gives a NOT-OR gate (or simply NOR gate). The symbolic representation for NOR gate is given in figure





Truth table of NOR gate

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0



Circuit diagram for NOR gate

NAND and NOR as the basic building blocks:

NAND and NOR gates are considered as universal gates because you can obtain all the gates like AND, OR, NOT by using NOR and NAND gate.

Figure shows the construction of NOT, AND and OR gates using NAND gates.

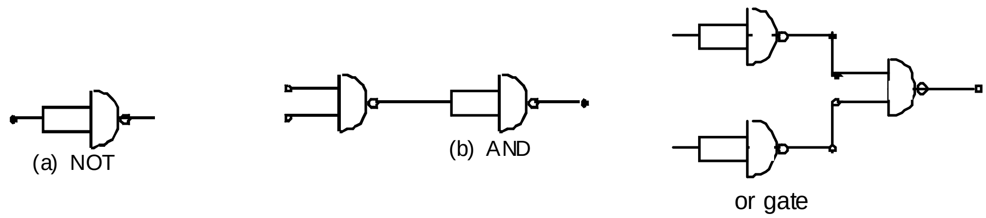
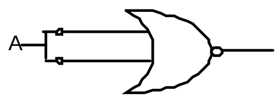


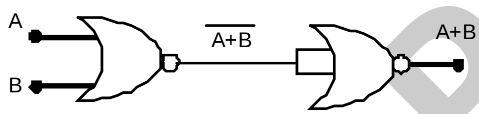
Illustration 3. *NOR gate is a universal gate. Prove that NOR gate can be used to realize OR, AND and NOT gates.*

Solution: As NOT Gate



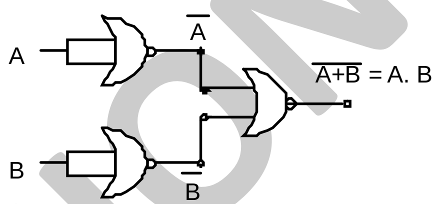
outputs obtained are just inversions of inputs. Which is Boolean expression for NOT gate.

As OR Gate



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

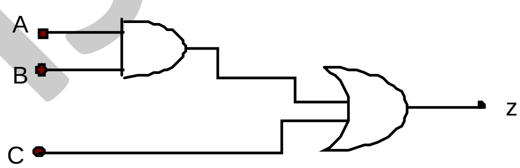
out put Y is same as of OR gate



A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Output is same as of AND gate.

Illustration 4. *Find the boolean equations for the output of the logic circuit shown in figure.*



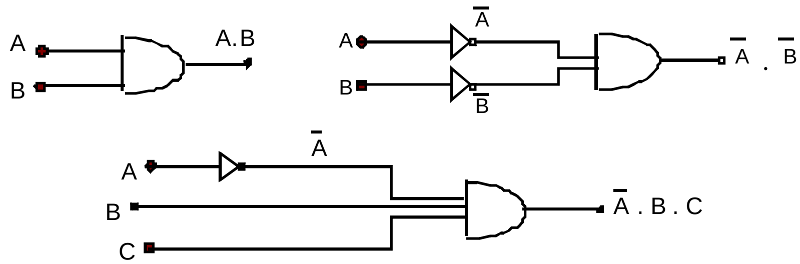
Solution: A and B are ANDed so their output is B. It then becomes one of the inputs for the 2-input OR gate. When AB ORed with C.

z = AB + C

Illustration 5. *Draw the logic circuit represented by the expression*

$$z = AB + \overline{AB} + \overline{A}BC$$

Solution: In the given expression there are three input logical variables and z is the output.



- The first terms AB is obtained by ANDing A with B.

- The second term $\bar{A}\ \bar{B}$ is obtained by 2 inverters and one AND gate.

- The last term is obtained by using one INVERTER and one AND gate and connecting them.

Now the complete logic expression is realized by ORing the three outputs.

SUMMARY

- A *p-n* junction diode can be used as a rectifier to convert ac into dc.
- A half-wave rectified dc contains more ac component than the full-wave rectified dc.
- A Zener diode stablizes the output of a power supply.
- In a stabilizer, the Zener diode dissipates more power when the current taken by the load is less.
- For amplification, a transistor needs input current.
- Transistor can be used as a switch by biasing it into saturation and cut-off regions.
- There are three basic logic gates: AND,OR and NOT.
- NAND gate is a universal gate because it can be used to implement other gates easily.

FINAL EXERCISE

1. Why the Peak Inverse Voltage (PIV) of a *p-n* junction diode in half-wave rectifier with filter capacitor is double of that without the capacitor?

2. Explain how a Zener diode helps to stabilize dc against load variation.

3. What should be the range of variation of amplitude of input signal for proper working of an amplifier?

4. Draw a circuit using diodes and transistors to implement a NOR gate.